MODELLING OF PERFORMANCE OF AN ARTILLERY SHELL USING NEURAL NETWORKS

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CERTIFICATE

It is certified that the work contained in this thesis entitled, **"Modelling of Performance of an Artillery Shell using Neural Networks"** by Santosh Kumar Dehury has been carried out under our supervision and that this work has not been submitted elsewhere for a degree.

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ABSTRACT

In recent times, the most widely used trajectory modelling for artillery shells has been via mathematical models such as point mass model, modified point mass model and six-degrees-of-freedom model. Applications of these models require an a priori postulation of equations of motion governing the shell trajectory and to solve these equations, one needs reliable estimates of aerodynamic coefficients. Due to various assumptions used in arriving at the mathematical models and also due to non availability of reliable estimates of aerodynamic coefficients required to solve these equations, an alternative approach of using general function approximation capability of the feed forward neural networks (FFNNs) for estimating shell performance is explored. In the literature, no work has been reported wherein FFNNs have been used to develcp neural models for shell performance. The present work addresses this aspect by way of proposing three distinct neural models for predicting trajectory variables. The models are validated for 155 mm Bofors shell HE77B data supplied in the form of range tables by ARDE, Pune. The estimated trajectory parameters like the range, the firing angle, the time of flight, etc., via the proposed neural models compare well with those listed in the supplied range tables. The neural models developed take into account variations in shell mass and its muzzle velocity, and variable atmospheric conditions like head/tail wind, crosswind, temperature and density that might prevail at the time of firing. It is shown that the proposed models can accurately predict i) the range obtainable for varying firing angles under prevailing atmospheric conditions, ii) the firing angle required to achieve desired range for known atmospheric conditions, and iii) the standard range that the shell would have achieved under standard atmospheric conditions.

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Santosh Kumar Dehury

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NOMENCLATURE

BAT	= Ballistic Air Temperature (ambient air temperature), K
BAD, ρ	= Ballistic Air Density (ambient air density), kg/m^3
PW, m	= Projectile mass, kg
θ	= Firing Table Elevation, Firing angle, mils
Ψ	= Bearing Correction for Drift, mils
T, t	= Time of flight, sec
d	= Diameter of Shell, mm
W	= Wind Speed, m/s
C_L, C_{Do}, C_M	= Non-dimensional lift, drag and pitching moment coefficient
u	= Projectile velocity, m/s
v	= Projectile air-relative speed, m/s
α_r	= Yaw of repose
ω	= Rotation vectors
Ω	= Earth rotation angular velocity, rad/s
g	= Acceleration due to gravity, m/s^2
I_x, I_y	= Moment of inertia about x and y axis, kgm^2
P	= Roll rate, spin rate of projectile, rad/s
q	= Pitch rate, rad/s
R	= Radius of earth, m
r	= Distance from earth center to C.G. of the shell
α	= Angle of attack
h	= Angular momentum

Superscripts

$\rightarrow$	= vector quantity
.	= Derivative with respect to time

Subscripts

r	= repose
x,y,z	= component along x, y and z direction

Non-dimensional stability derivatives

$$C_{L_\alpha} = \frac{\partial C_L}{\partial \alpha}; \qquad C_{L_q} = \frac{\partial C_L}{\partial\left(\frac{qd}{2v}\right)}; \qquad C_{L_p} = \frac{\partial C_L}{\partial\left(\frac{pd}{2v}\right)}$$

$$C_{L_{\dot\alpha}} = \frac{\partial C_L}{\partial\left(\frac{\dot\alpha d}{2v}\right)}; \quad C_{y_{p\alpha}} = \frac{\partial C_y}{\partial p\alpha}; \qquad C_{D_{\alpha^2}} = \frac{\partial C_D}{\partial \alpha^2}$$

$$C_{M_\alpha} = \frac{\partial C_M}{\partial \alpha}; \quad C_{M_{p\alpha}} = \frac{\partial C_M}{\partial p\alpha}; \quad C_{M_q} = \frac{\partial C_M}{\partial q}$$

CHAPTER 1
INTRODUCTION

Artillery forms an important wing of the army to provide firepower during war as well as during cross-boarder skirmishes with the enemy. Artillery generally falls into three basic categories: guns, howitzers and mortars. The principle difference between them being the trajectory of the round fired. A gun has a high muzzle velocity and a very flat trajectory. Normally a gun is used in a direct fire mode where the target can be seen and penetration is desirable. Howitzers have a somewhat lower muzzle velocity and arc their shells onto a target. They are used in both a direct fire and indirect fire mode. This is especially useful when an enemy is concealed behind a prepared position or the artillery men desire to have a shell explode over an enemy's head. The air-burst does less damage to hardened targets, but causes many more human casualties due to shell fragmentation covering a large area. Mortars have a very pronounced arc of flight. They have a relatively low muzzle velocity and are unsuitable for direct fire. Their principle value comes from being able to lob shells behind an obstacle, such as a fortification or a hill. They are not very accurate and dependent upon the amount of propelling powder to determine the point of impact.

The effectiveness of artillery is largely judged by the accuracy in hitting the targets. The accuracy and reliability is influenced by the design criteria used in designing the shell. Artillery shells are a class of projectiles around which much of the aeroballistic theory was originally developed, and they continue to form a significant part of the aeroballistician's interest.

Study of the motion of projectiles through an external medium is known by the name of external ballistics. If the external medium is the earth's atmosphere, then external ballistics become synonymous with aeroballistics.

Each specific projectile will have its own individual design criteria with regard to payload, range, accuracy, etc. The practical design criteria applying to a conventional artillery shell which most affect the aeroballistic aspects are:

1) High payload carrying potential
2) Range at high charges (muzzle velocity around Mach 2-3) must be maximized - hence low drag
3) Accuracy and repeatability
4) Ease of manufacturing.

The accuracy of artillery shell depends on the following major factors:

1) Muzzle velocity irregularity due to variation in charge
2) Jump and throw-off: This occurs due to recoiling effect of gun. Due to this action, there is a vertical component, known as jump, and there is a horizontal component of that force known as throw-off.
3) Meteorological effects like ambient temperature, density, head/tail wind, crosswind, etc.
4) Differences between shells due to shape, size, mass, etc.
5) Difference in yawing behavior, at the time of exit from the barrel, due to muzzle blast and wind consideration.

The first two factors are outside the control of the aeroballistician. The rest can be minimized by ensuring that the shell is both adequately gyroscopically and dynamically stable under all launch conditions, and has low drag.

The conventional approach hitherto for understanding the in-flight behaviour of projectiles was to develop mathematical models that could predict all elements of the trajectory from launch to target. To this purpose, it becomes essential that all forces, moments and other terms (e.g., coriolis force due to the rotation of the earth) affecting the flight of the projectile are accounted for in a well defined mathematical form[1]. Beginning with the most simple but relatively inaccurate mathematical model, the in-vacuo trajectory model, more and more sophisticated models of increasing accuracy such as the point mass model, the modified point mass model and the six-degrees-of-freedom model have been developed. A brief description of these models is given in chapter 3. Presently, we only wish to point out that even the best of these proposed models have their limitations due to their inability to model all the problem variables adequately. For example, the initial conditions at the time of shell leaving the barrel are not accounted for by any of the proposed mathematical models. Furthermore, the models require aerodynamic coefficients as input (e.g., drag coefficient, damping in roll derivative C_{lp}, lift curve slope $C_{L\alpha}$, etc.) and the estimates available for these coefficients are not so reliable. Finally, the varying atmospheric conditions over the height traversed by the projectile are accounted for only in an adhoc manner by using one single weighted mean value of the variables like temperature, density, head/tail wind and crosswind.

It is thus realized that even the best of the mathematical models available to date are not reliable for field application because accurate predictions are difficult for: 1) the range obtainable for various firing angles, 2) the firing angle required for specified range.

The limitations of the mathematical models so far used for predicting the performance of artillery shell motivated us to look at an alternative approach to modelling. The feed forward neural network[2,3,4] provides one such potential way of modelling.

The neural networks have been successfully used in such diverse fields as signal processing, pattern recognition, system identification and control. In recent years, neural models of aircraft aerodynamics have been successfully developed for many applications relevant to Aerospace Engineering[5,6,7]. For example, neural modelling has been used for estimating aircraft stability and control derivatives of stable[8,9,10], unstable[11], aeroelastic aircraft[12]. It was envisaged that a neural model can be developed to replace the use of hitherto used mathematical models for solving the shell trajectory related problems.

For real life situations, it was realized that the problem of shell trajectory, broadly speaking, could be divided into three categories: Under prevailing ambient atmospheric conditions 1) to predict range for the chosen firing angle, θ, 2) to predict required firing angle to achieve specified range, 3) to predict range that would have been achievable under standard atmospheric conditions, using the range data obtained under existing atmospheric conditions. The next step is to identify the relevant set of input-output variables for the network for each of these problems. Finally, a suitable architecture for the Neural Networks is to be searched to achieve acceptable functional mapping between the input-output variables for each of the problem.

In the present work, all the above problems are addressed and adequately solved. For demonstrating the prediction capability of neural models developed, the data used is for the 155mm Bofor shell, supplied by ARDE, Pune. Details of the data supplied and its use for testing various neural models is given in chapter 3. The details of modelling for the three categories of problems related to the shell trajectory, along with results obtained from each of three models are discussed in chapter 4. A brief overview of the neural network precedes these chapters 3 and 4, and is given in chapter 2. The dissertation ends with chapter 5 containing conclusions and a few suggestions for the future work.

CHAPTER 2

ARTIFICIAL NEURAL NETWORK

2.1 Introduction

A neural network is a parallel distributed processor that has a natural propensity for storing experimental knowledge and making it available for use. It resembles the brain in two respects, 1) Knowledge is acquired by the network through learning process, 2) Inter neuron connection strengths known as synaptic weights are used to store the knowledge. Neural networks are also referred to in the literature[2,3,4] as neuro computers, connectionist networks, parallel distributed processors, etc.

Artificial neural networks consist of group of neurons arranged in a layered structure. Each neuron receives signal from the neurons in the layer previous to itself and passes a signal on to the neuron in the following layer. The relationship between the summed inputs to a neuron and its output is governed by an 'activation function'[3]. Some of the commonly employed activation functions are the step functions, the tangent-hyperbolic function, and the logistic (sigmoidal) function.

Out of these, the most often-used activation function is the sigmoidal function defined as

$$F(x) = \frac{1}{1+e^{-x/\lambda}}$$

where λ is the logistic gain.

The sigmoidal function is continuous, monotonically increasing and continuously differentiable, and it asymptotically approaches fixed finite values as the input approaches infinity (+ or -).

Amongst the artificial neural networks, the feed forward neural networks (FFNNs) have found the favour with most researchers for applications in aerospace engineering problems[5,6,7]. The feed forward neural network consists of source nodes that constitute the input layer and one or more hidden layers and an output layer. Feed forward neural networks have neurons arranged in layers like directed graphs, implying a unidirectional flow of signals, and thus are static in nature. This kind of modeling develops input-output relationship of a black-box kind. Each of the connection between neurons is assigned its individual weight and it is adjusted so as to yield the required output corresponding to the known set of inputs. Assignment of weights is done during the training sessions of the network.

2.2 Back Propagation Algorithm

One of the efficient methods for training the FFNN is the back propagation algorithm (BPA). Back propagation algorithm consists of a forward and backward pass through different layers of the network. During the forward pass, the input vector is applied to the input nodes of the network and its effect propagates through the network layer by layer. The connective weights are all kept fixed during the forward pass. On the other hand, in the backward pass, the weights are updated in accordance with error correction rule. Specifically, the actual response of the network is subtracted from the desired response to produce

an error signal. This error signal is then propagated backward against the direction of the connective weights. There are the four steps for back propagation algorithm.

a) Initialization: Start with a reasonable network configuration and set all the synaptic weights randomly.

b) Forward Computation: Let a training example represented by x(n) be applied to the input nodes. Then the output of the network is computed by proceeding through the network, layer by layer. Next, the error signal is computed using difference between the desired response vector and the output vector.

c) Backward Computation: The back propagation algorithm is based on optimizing a suitably defined error function. At each point, the local output error cost function defined by the sum of the squared error is computed. The weights of the network are adjusted in such a way that the mean squared error (MSE) is minimized.

The MSE is given by

$$\mathrm{MSE} = \frac{1}{m \times n} \sum_{i=1}^{m} \sum_{j=1}^{n} [Yi(j) - Xi(j)]^2$$

where Y and X are the desired and predicted outputs, n is the number of data points and m is the number of output variables.

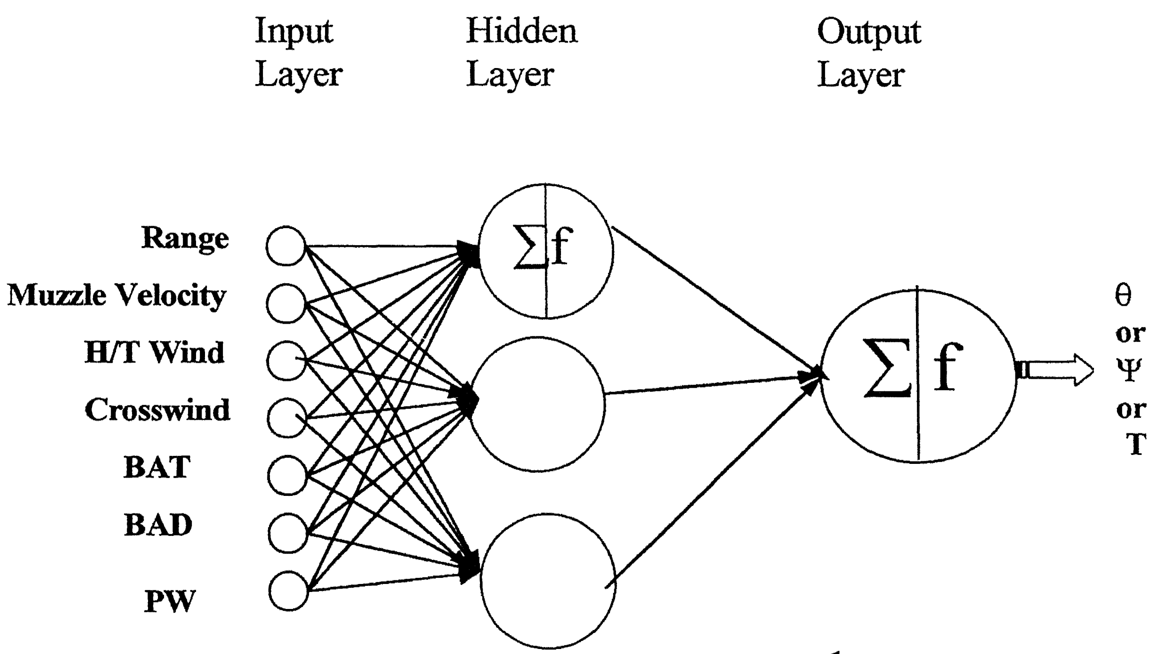
d) Step (b) to (c) are repeated for each training pair in the training set until the error for the entire set is less than the prescribed value or the number of iterations exceed the prescribed limit.

The training algorithm is recursive in nature and it needs repetitive training sessions to achieve the required learning. There are many network influential (tuning) parameters like the learning rate, the momentum rate, the number of hidden layers, the number of iterations, the logistic gain, etc., that affect the accuracy of functional mapping between the input and the output variables. There are no set rules for fixing values of these influential parameters. In literature, a few guidelines are available to guide the choice of these parameters. However, the final choices of fine tuning these parameters is to be achieved by trial and error for the given problem, and it is a crucial step in finding a suitable neural model for the problem. Some of the guidelines and thumb rules for training of neural network are given in Appendix B.

The neural network model is first trained on the known sets of input-output pairs of experimental or recorded data. The trained network is then capable of predicting the required output based on known (measured) input variables. This approach does not require a mathematical model or a transfer function relating the input-output data but only a sufficient set of input-output pairs of data. The choice of inputs that are likely to affect the output to be predicted by the neural model is of critical importance. The selection of inputs for the given problem is based on the understanding of the physics of the problem and engineering judgement.

In the present work, a neural model is proposed for the trajectory modelling of an artillery shell fired from a gun. A non-linear relationship is mapped by the neural network between the input variables such as range, muzzle velocity, wind velocity, ballistic air temperature (BAT), ballistic air density

(BAD) with the output variables such as time of flight (T), bearing corrections for drift (Ψ) and angle of firing (θ). A schematic of such a neural model is shown in fig.1. For a typical neural model used in the present work, Fig.1, the choice of the input and output variables for mapping projectile dynamics is dictated by the physical understanding of the phenomenon governing the shell dynamics. Once the input and output variables for the feed forward artificial network (FFNN) are selected, the FFNN model of shell dynamics is achieved without the need of a formal model structure formulation.



$$f = \frac{1}{1+e^{-x/\lambda}}$$

Fig. 1 Schematic of feed forward neural network.

CHAPTER 3

TRAJECTORY AND RANGE MODELLING OF ARTILLERY SHELL

3.1 General

Artillery shells are a class of projectiles around which much of the aeroballistic theory was originally developed. Even today, it still continues to be of interest and further investigations for many aeroballisticians and user agencies. In the present work, we are specifically to focus on flight of a spin stabilized, dynamically stable, conventional artillery projectile, possessing either rotational or some form of mirror symmetry. For such projectiles, there are two major factors that differentiate their aerodynamics from the classical aerodynamics of, say, aircraft aerodynamics:

1) The geometric symmetry of the projectile implies that many aerodynamic terms are themselves symmetric. For example, for a shell with cruciform tails will have aerodynamic derivatives $Cm_{\alpha} = -Cn_{\beta}$ and $Cm_q = Cn_r$.
2) The second factor to be considered is due to spin rate imparted to many shells, giving rise to aerodynamic effects that are unique to aeroballistics. For example, the magnus forces and moments are hardly of concern for aircraft dynamics, but may play significant role for shell dynamics.

Presently, the conventional approach to modelling of trajectory and range of artillery shell has been by postulating a suitable mathematical model consisting of equations of motion. There are numerous such forms of trajectory model involving different basic assumptions and having different complexities of solution. We shall briefly outline, in an increasing order of complexities and accuracy, the more commonly utilized analytical trajectory models used at present time.

3.2 The in-vacuo trajectory model

In the in-vacuo trajectory model all aerodynamic forces and moments acting on the projectile are neglected. It is thus highly inaccurate for all but the shortest-range trajectories and the lowest drag projectiles. However, it does have its uses, in defining concepts such as trajectory rigidity, and also for setting limits on parameters such as range and vertex height. The simplicity of the model renders analytical solution easily.

3.3 The point mass model

In this model, it is assumed that the only aerodynamic force acting on the projectile is drag. It provides fairly accurate estimates of range for adequately stable projectiles and can also be used to estimate the first order effects of wind. The point mass model is given as below.

$$\frac{d^2x}{dt^2} = -\frac{\pi\rho d^2 C_D}{8m} v\left(\frac{dx}{dt} - W_x\right) \tag{1a}$$

$$\frac{d^2y}{dt^2} = -g - \frac{\pi\rho d^2 C_D}{8m} v\left(\frac{dy}{dt} - W_y\right) \tag{1b}$$

$$\frac{d^2z}{dt^2} = -\frac{\pi\rho d^2 C_D}{8m} v\left(\frac{dz}{dt} - W_z\right) \tag{1c}$$

where x denotes range, y denotes height, z denotes drift and W_x, W_y, W_z are, respectively, the x, y and z components of the wind velocity W. This model, however, does not account for the effect of spin of the shell and thus fails to predict the drift due to magnus moment. Also, it neglects the lift forces acting on the shell.

3.4 The Modified point mass model

The modified point mass model is also known as 4 degree-of-freedom model (3 spatial degrees-of-freedom plus axial spin). Its basis is a conventional point mass, in addition, the instantaneous equilibrium yaw is calculated at each time step along the trajectory so as to provide estimates of yaw, drag, drift and magnus force effects resulting from the yaw of repose. The modified point mass equations of motion are given as

$$\frac{d\vec{u}}{dt} = -\frac{\pi\rho d^2}{8m}\left(C_{D_o} + C_{D_{\alpha^2}}\alpha_r^2\right)v\vec{v} + \frac{\pi\rho d^2}{8m}C_{L_\alpha}v^2\alpha_r -$$

$$\frac{\pi\rho d^3}{16m}C_{y_{p\alpha}}p(\alpha_r \times \vec{v}) - g_o\frac{R^2}{r^3}\vec{r} + 2(\omega \times \vec{u}) \qquad (2a)$$

$$\frac{dp}{dt} = \frac{\pi\rho d^4}{16 I_x}pvC_{l_p} \qquad (2b)$$

$$\alpha_r = -\frac{8pI_x}{\pi\rho d^3 C_{M_\alpha}}\frac{[\vec{v} \times (d\vec{u}/dt)]}{v^4\alpha_r} \qquad (2c)$$

The quantities in the above equation are defined as follows:

$\vec{u} = \frac{d\vec{x}}{dt}$, $\vec{v} = \vec{u} - \vec{W}$, $\vec{r} = \vec{x} - \vec{R}$, $\vec{R} = (0, -R, 0)$ [$\vec{W}$ denotes the wind vector]

$\vec{\omega}$ = ($-\Omega$cos[latitude]cos[azimuth],

Ω sin[latitude],

Ω cos[latitude] sin[azimuth])

where $\Omega = 7.29\times10^{-5}$ rad/s (rotation of the earth),

$R = 6370320$ m (radius of the earth),

$g_o = 9.80665[1-0.0026373 \cos(2\times\text{latitude}) + 0.0000059 [\cos(2\times \text{latitude})]^2]$.

The axis system used is as for the point mass model with $\vec{x}$ being along the line of fire.

As may be seen from the above equations, the aerodynamic coefficient input required is extensive and accuracy of these aerodynamic coefficients is crucial for reliable estimates of range. However, for most artillery shells, the reliability of estimated values of aerodynamic coefficients is not high enough to inspire confidence in resulting range estimates from this model. In spite of this limitation, this model has the capability of providing reasonable accounts of end point data. It can estimate wind corrections and can accept non-linear aerodynamic inputs if required. However, because the equation for calculating yaw of repose assumes quasi-linear aerodynamics, the use of non-linear aerodynamics in equations is questionable.

3.5 The six-degree-of-freedom model

As a final step, the most sophisticated trajectory model developed was the six-degree-of-freedom model having the 3 spatial degrees of freedom, yaw degree of freedom in two planes and the spin. In this model there are no assumptions concerning linearised aerodynamics or projectile symmetry. However, the indeterminability of many of the initial conditions and aerodynamic coefficients which are required as input frequently results in the model not giving significantly better end results than the modified point mass model. Thus its usage might not be justified for routine fire control work. It is nevertheless a powerful tool for the ammunition designer.

The six-degree-of-freedom model equations are given as

$$\frac{d\vec{u}}{dt} = -\frac{\pi \rho d^2}{8m}\left(C_{D_0} + C_{D\alpha^2}\alpha^2\right)v\vec{v} \qquad \text{(Drag)}$$

$$+\frac{\pi \rho d^2}{8m} C_{L_\alpha} \left[\vec{v} \times (\vec{x} \times \vec{v})\right] \qquad \text{(Lift)}$$

$$+\frac{\pi\rho d^3}{16m} C_{y_{p\alpha}} \frac{I_y}{I_x} (\vec{h}\cdot\vec{x})(\vec{x}\times\vec{v}) \qquad \text{(Magnus Forces)}$$

$$+\frac{\pi\rho d^3}{16m}\left(C_{L_q}+C_{L_{\dot\alpha}}\right)v(\vec{h}\times\vec{x}) \qquad \text{(Damping Forces)}$$

$$-g_0\frac{R^2}{r^3}\vec{r}+2(\vec{\omega}\times\vec{u}) \qquad \text{(Gravity and Coriolis)} \qquad (3a)$$

$$\frac{d\vec{h}}{dt}=\frac{\pi\rho d^4}{16I_x}C_{l_p}(\vec{h}\cdot\vec{x})v\vec{x} \qquad \text{(Spin Damping)}$$

$$+\frac{\pi\rho d^3}{8I_y}C_{M\alpha}v(\vec{v}\times\vec{x}) \qquad \text{(Overturning Moment)}$$

$$-\frac{\pi\rho d^4}{16I_x}C_{M_{p\alpha}}(\vec{h}\cdot\vec{x})[\vec{x}\times(\vec{x}\times\vec{v})] \qquad \text{(Magnus Moment)}$$

$$+\frac{\pi\rho d^4}{16I_y}\left(C_{M_q}+C_{M_{\dot\alpha}}\right)v[\vec{x}\times(\vec{h}\times\vec{x})] \qquad \text{(Damping Moment)} \qquad (3b)$$

where

$$\alpha=\cos^{-1}[(\vec{v}\cdot\vec{x})/v]$$

and $I_y\vec{h}$ is the angular momentum vector.

3.6 Range Tables for B-Shell

As mentioned earlier, due to non availability of real firing data, the range tables for the 155mm Bofors shell HE 77B (here after referred to as B-shell for convenience) are used for the present study. These tables and the explanatory notes (given in Appendix A) were supplied by ARDE, Pune. The range table lists range obtainable for various firing table elevations (hereafter referred to as firing angle, θ) under standard calm atmospheric conditions and for nominal values of weight and muzzle velocity. Also time of flight and correction to bearing drift is provided. The corrections to these listed values of range for each value of θ due to following variations at the time of firing are also provided:

1) Variations in ambient atmospheric conditions (temperature, density)

2) Head/tail wind

3) Crosswind

4) Change in weight and muzzle velocity from the corresponding nominal value.

The range tables have been prepared using the modified point mass trajectory model. As pointed out earlier, this model suffers from a few limitations, mainly because of the not-so-reliable accuracy of aerodynamic coefficients that need to be used to solve the equations of motion. The complete six-degree-of-freedom model would also suffer for the same reasons. Even if accurate aerodynamic coefficients were available, there are few other uncertainties that would affect the range estimates in real life and cause dispersion of shells. A few such uncertainties are briefly outlined below:

1. Slight differences between nominally similar projectiles

Any differences in mass, shape, surface finish, etc., will cause changes in the trajectory of the projectile to some extent. Projectiles might suffer from two types of asymmetries while passing through the manufacturing stage: Configurational asymmetries and inertial asymmetries. The inertial asymmetries may be of mass unbalances wherein center of mass is not on the geometrical axis of symmetry and dynamic unbalance where the principal axis of inertia is not collinear with this axis. These asymmetries can lead to large dispersions, and it is by keeping manufacturing tolerances to minimum, and by imparting some amount of spin that one can reduce the dispersion due to this effect. However, spin rate is to be carefully chosen to avoid the spin-yaw resonance zone.

2. Meteorological conditions

It is noted that the solution of equations of motion uses a single value for head or tail wind and crosswind. Furthermore, the corrections to the range (tabulated for standard atmospheric conditions) for variation in temperature and density are incorporated by using a single value of temperatures and density at the location of firing. It is realized that the shell will be passing through different wind and atmospheric conditions of temperature and density as its altitude varies during its in-flight trajectory. However, for the purpose of applying corrections, a weighted mean value of wind velocity, temperature and density is used to account for varying conditions prevailing at different altitudes.

3. Variable launch conditions: Variation in muzzle velocity, gun-jump and throw-off, initial yaw, etc.

At the time of shell leaving the gun barrel, the initial conditions experienced by the shell are not identical for all the shells. In particular, the initial velocity of shell (muzzle velocity) depends upon the charge concentration at the time of firing.

For the spun projectiles crosswind has an effect in the vertical plane due to the time taken to adapt to the air-relative zero yaw position. This occurs whenever the projectile emerges into a region of significantly different wind-relative zero yaw attitude, but most commonly has its effect on a projectile exiting from the launcher. Unlike the downward carry effect, which is usually roughly quadratic in range and depends on the wind at all points down-range, this effect is called aerodynamic jump, which is linear in range and dependent on wind at one point. There is a similar term due to the transverse component of a head wind. The total effect is called windage jump.

The initial yaw and yawing rate of a projectile will obviously determine to a large extent what yaw levels are present in the early stage of the trajectory. Furthermore, if the damping is poor, they may lead to consistently high yaw and thus cause dispersion in range due to yaw drag, especially with indirect fire munitions.

3.7 The Neural Models

The limitations of the mathematical models so far used for predicting range of artillery shells motivated us to look at an alternative approach to modelling. The feed forward neural network provides one such potential way of modelling. The neural model needs identification of suitable input-output variables to map the relationship existing between them. The functional relation is obtained by training with measured input-output variables and then the trained network is used to predict the output for set of inputs not seen by network during the training phase. The data from the range table provided by ARDE, Pune for B-shell is used for the present work.

For the purpose of developing a neural model, the data from the range table was randomly selected to form sets having chosen number of data points, say, 100 or 50 or 25 or 15. One of these sets is used for training the network. Then two sets of data points, each of 30 are taken for validation sets, which are different from the training set. These two sets are used to verify the acceptability of the trained network. As mentioned in Appendix B, the acceptability of the network architecture is decided by comparing the MSE during training phase and validation phase. The thumb rule applied is that MSE for the two validation sets should be not greater than about two times the MSE for the training set. Of course, the MSE permitted on the training set is prescribed and kept below it, while choosing the architecture of the network, i.e., while choosing the learning

rate, the momentum rate, the number of iteration, the number of neuron, etc. Finally, one set of data, not used for the above two stages of training or validation is used to predict the required output. This output is compared with the known output corresponding to the same inputs, and thus shows acceptability of the neural model in predicting the required output variables.

From application point of view, the following three types of modelling problems were taken up for study. The neural network modelling for these three cases are presented in details in the next chapter along with the results obtained via each of these models. However, a brief outline of the three models is given below.

Model 1

This model deals with the direct problem of modelling Range, time of flight, and drift experienced by the B-Shell. These output variables of the neural model would depend on the firing angle θ, air temperature and density, head/tail wind, crosswind, muzzle velocity, weight of shell and the initial conditions.

Model 2

This model deals with the inverse problem of predicting the firing angle θ required to achieve desired range, given the air temperature and density, head/tail wind, crosswind, muzzle velocity, weight of the shell. It also predicts time of flight and drift. This is what a user (soldier) would require to know in most of the real life applications.

Model 3

This model was developed to predict the range that would be obtainable under standard conditions, given the measured data of range obtained under varying

atmospheric conditions like temperature, density, head/tail wind, crosswind, muzzle velocity, and shell weight. Such a prediction capability would be useful to compare performance of different shells under identical (standard) conditions. It is not practical to obtain data for different shell or for the same shell on different days under similar conditions, and thus compare the relative performance. If data collected under different ambient conditions can be used for training the network, and then be able to predict the range that would result under chosen standard ambient conditions, one could then compare results for the different shells or the same shell fired at different times or days.

As a subset of this model, attempt was also made to estimate sensitivity coefficients (partial derivatives) that would show how the range is affected by variations in each of input variables. These sensitivity coefficients were in turn used to obtain standard range from the range data corresponding to ambient conditions of temperature, density, wind, etc.

The results and discussions for all the above three models, along with details of modelling are presented in the next chapter.

CHAPTER 4

NEURAL MODELS, RESULTS AND DISCUSSION

In this chapter, we first analyse the data of B-shell supplied by ARDE, Pune. Next, details of all the three models mentioned in the previous chapter are presented. Finally, the results for all the neural models are discussed.

4.1 Analysis of B-shell data

The data sets, explanatory notes and geometric, mass and moment of inertia characteristics of B-shell supplied by ARDE, Pune are given in Appendix A. The data tables (Table F (i) and Table F (ii)) contain range data for firing table elevation (Firing angle) from 0 mil to 1298.4 mils. These firing angles correspond to variation from 0° to 70° ($360^{\circ} = 6400$ mils). Table F (i) has the basic data for range (X, col 1) along with the time of flight (t, col 7), correction to bearing for drift due to spin ($\Delta_C A_d$, col 8) and correction to bearing for 1 knot crosswind ($\Delta_C A_B$, col 9) obtainable for varying firing angles (A_E, col 2) at sea level under standard atmospheric conditions. Tables F (ii) gives corrections to range for the non-standard conditions. Corrections to be applied to calculate the range at various firing angles for variation from the nominal values of muzzle velocity, projectile weight, air temperature and density, and for mean weighted head/tail wind are tabulated. In Table F (ii), column 2 gives correction for muzzle velocity being 1m/s higher or lower than nominal value, column 3 gives correction for 1knot head/tail wind, column 4 and 5 give, respectively, corrections for 1% higher or lower value of temperature and density as compared to those for standard atmosphere, column 6 gives correction for weight of shell being 1 unit (1 unit = 0.45 kg) higher or

lower than its nominal value. The nominal values of the B-shell and its geometry are given in fig. A1 of Appendix A. Explanatory notes on the terminology used in Range table is given following the range table in Appendix A.

The range table F (i) and F (ii) were used to prepare input-output data sets for the neural models. The range R for non-standard conditions was calculated as follows:

$$R = X + C_1 (\Delta V) + C_2 (W_x) + C_3 (\%\ BAT) + C_4 (\%\ BAD) + C_5 (\Delta PW) \qquad (4.1)$$

where X = Range at standard condition for a particular firing table elevation (firing angle, θ).

C_1 = Correction to range for muzzle velocity being higher or lower by 1m/s than the nominal velocity, given in column 2 of the table F(ii) of appendix A., sign of C_1 is negative if velocity lower and positive if velocity is higher than the nominal value.

ΔV = Muzzle velocity - nominal value of 818 m/s.

C_2 = Correction to range for 1 knot head/tail wind, shown in column 3 of table F(ii) of Appendix A, sign is negative for head wind and positive for tail wind.

W_x = Weighted mean value of head/tail wind.

C_3 = Correction to range for temperature being 1% higher or lower than value in standard atmosphere, given in column 4 of table F(ii) of appendix A, sign is negative for temperature being higher and positive for it being lower than standard value.

% BAT = (Actual BAT - BAT in Standard atmosphere) × 100/Actual BAT

C_4 = Correction to range for density being 1% higher or lower than value in

standard atmosphere, given in column 5 of table F(ii), sign is negative for density being higher and positive for it being lower than standard value.

% BAD = (Actual BAD - BAD in Standard atmosphere) $\times$ 100/Actual BAD

C_5 = Correction to range for 1 unit change in projectile weight, given in the column 6 of the table F(ii) of Appendix A, sign is negative for weight being higher and positive for being lower than the nominal value of 42.6 kg

ΔPW = (Actual weight of Shell - 42.6)

It is noted that the crosswind gives rise to drift in the lateral direction but does not affect the range.

To illustrate the calculation of range R for non-standard conditions, the following example is presented:

Consider the range for firing angle of 67.2 mils. The Table F(i) gives the standard range X = 6600 m, the time of flight = 10.2 sec, correction to bearing for drift $\Delta_C A_d$ = 2.8 mils and correction to bearing for 1 knot crosswind $\Delta_C A_B$ = 0.17 mils.

Let us assume that the muzzle velocity is 819.9 m/s, tail wind is 8.01 knots, crosswind is 8.7 knots, projectile weight is 43.1769 kg, air temperature and density are 297.381K and 1.2676 kg/m^3 respectively. For the firing angle of 67.2 mils, from table F(i) and F(ii), we have

C_1 = -13.0 m / m/s

C_2 = -1.7 m / knot

C_3 = 4.4 m / 1% change in BAT

C_4 = 17.9 m / 1% change in BAD

C_5 = 13 m / 1unit change in projectile weight

For the chosen values, the following quantities are easily calculated

ΔMuzzle velocity = ΔV = 1.9 m/s,

ΔProjectile weight = ΔPW = 1.282 unit,

% change in BAT = 3.2,

% change in BAD = 3.479,

Substituting above value in Eq. (4.1), we get the non-standard range:

$$R=6600 - [(-13.0 \times 1.9) + (-1.7 \times 8.01) + (4.4 \times 3.2) + (17.9 \times 3.479) + (13 \times 1.282)]$$

$$= 6545.3 \text{ m.}$$

Bearing Correction (Ψ) = Correction to bearing for drift + Correction to bearing for crosswind.

$$\Psi = 2.8 + (8.7 \times 0.17) = 4.279 \text{ mils}$$

The implication of the bearing correction is that the projectile would drift to the right (looking from behind in the direction of firing) by angle Ψ. To hit the target, the shell is fired by aiming to the left by angle Ψ and this would, in turn, ensure that the target is hit after the drift that the shell will experience due to spin and crosswinds.

Following the above procedure, the data given in the range tables for B-shell was converted into a convenient form for its use in neural models to be developed. Specifically, the data yielded sets of data, each set having standard range X, non-standard range R, time of flight T, drift angle Ψ, ambient temperature and density, crosswind, head or tail wind, variation from the nominal value of muzzle velocity and projectile weight, or percent variation thereof.

4.2 Modelling

The data sets generated from the range tables of B-shell were used for training the neural models. However, random selection of input-output pairs for any of the models (briefly described in chapter 3) showed poor training. A close look at the data and considering the physics of the projectile motion gave the clue: as is well known for idealized point mass projectile motion in vacuum, the range increases as the angle of projections increases from 0 to 45 degrees and then decreases as angle increases from 45 to 90 degrees, the maximum range being at 45 degrees. A similar trend was observable for the B-shell and maximum range was just above 45 degrees (802 mils). The implication of this fact is that same range is attained for two distinct values of angle of firing, one below 45 degree and other above 45 degree. Thus the neural network sees inherent contradiction in data and hence the difficulty in mapping the data spanned across all the values of firing angles.

To resolve the above difficulty, the data was partitioned into two bins: one having data for firing angle from 0 to 45 degree and the other from 45° to 70° (firing tables gives data up to 70° only). The neural models for these two were separately developed. It may be pointed out that, such an approach will not create any difficulty in real-life applications because the requirements about the desirable firing angle to be less than or more than 45 degree would be known - if the shell is to achieve higher altitude during its flight to target, the firing angle of greater than 45 degree would be recommended, otherwise less than 45 degree would be the preferred option due to shorter time of flight.

For the purpose of modelling, a set of randomly selected input-output data was selected from each bin. Sets of varying number of input-output samples were tried to arrive at the minimum number of samples required for adequate training level. It is realized that in real life, the number of samples available will be limited due to the cost involved in collecting such range data, and hence the need to search for the minimum number of data samples to get an acceptable neural model. The numbers of samples tried were 100, 50, 25 and 15. Higher number of samples obviously led to better training, but even as few as 15 samples also gave satisfactory models. The suitability of the model is tested by a test (validation) set of data, typically consisting of 30 input-output samples, selected randomly from the range table; if the MSE was only of the order of two times the MSE prescribed for the training phase, the neural model was accepted and its architecture fixed for the prediction phase. If not, the architecture was varied till it met the above conditions on MSE for training and validation phase.

Once the neural model is validated, it is used for the prediction purpose. For prediction, a set of randomly selected data is taken from the range table, and only the input variables of this set are treated as known while the output variable is predicted by the neural model. Since the output is also known from the range table, it is compared with the neural model output to show how well the predicted values compare with the known values. Typically, for prediction, 10 samples were randomly selected from the range table.

In case of neural models like Model 1 and Model 2, prediction of more than one output was required, e.g., Model 1 is to predict range, bearing correction and time of flight. In view of this, the neural models having more than one output were subjected to

the following test: for the same set of inputs, is it better to train the network separately for one output at a time, or train it on all the outputs at once. It was observed that even though the latter option gave reasonable predictions of all outputs, the former option always yielded relatively more accurate predictions. The accuracy being of prime concern, it is worthwhile to have separate neural models for each of the output variables and, therefore, all studies repeated herein are based on the single output predictions.

Because the above details of the procedure are common to all the models discussed next, we shall refrain from repeating it and only point out wherever the procedure differs in some significant way from the above mentioned guidelines. Results for the various models discussed next are presented in tabular as well as in graphical form.

4.3 Model 1

In model 1, we wish to develop neural model for predicting range(R), bearing correction (Ψ) and time of flight (T). These would, therefore, form the output variables of the neural model. These output variables depend on the firing angle (θ), ballistic air temperature (BAT), ballistic air density (BAD), muzzle velocity (V), projectile weight (PW), head/tail wind (W_x) and crosswind (W_z). Thus the neural model has R, or Ψ, or T as the output variable and θ, BAT, BAD, V, PW, W_x, W_z as the input variables. The input variables V, BAT and BAD are given in percent variation from the nominal values – nominal muzzle velocity being 818 m/s, and nominal BAT and BAD being sea level values in standard atmosphere. The ΔPW value was given as the difference between the actual projectile mass and the nominal projectile mass (42.6kg). The schematic of such a model is shown in fig. 2.

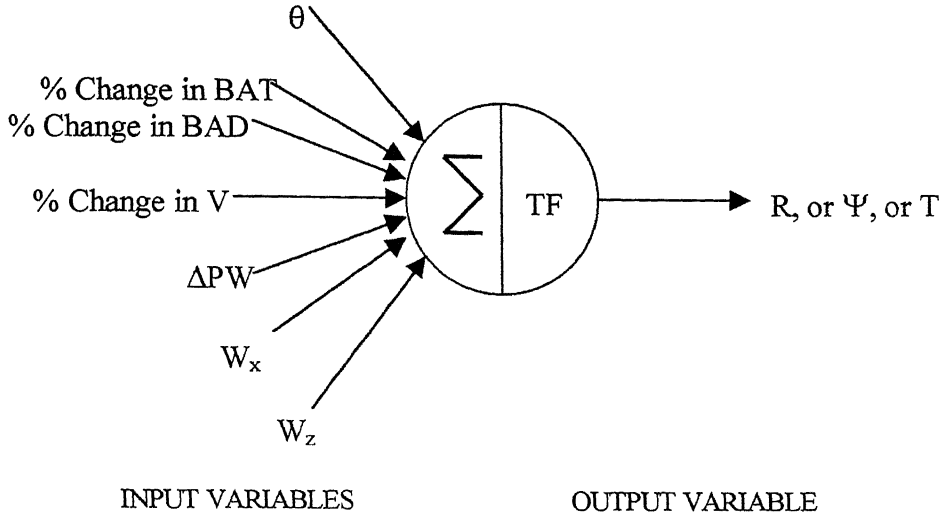


Fig. 2 Schematic Representation of FFNN For model 1

As mentioned earlier, input-output samples were randomly selected from the range table for B-shell. Although training was carried out for data sets having 100, 50, 25 and 15 samples, the results for the most stringent case of data set having 15 samples are presented. For data set having higher number of samples (> 15), the training was always superior to that for 15 samples. For illustration, MSE for 15 samples and 100 samples as a function of iteration is shown in fig.3. It is noted that the minimum MSE is achieved faster in case of 100 samples (in less than 200 iterations) as compared to 15 samples (in about 300 iterations). Furthermore, the value of minimum MSE achieved in case of 100 samples is about 8.22×10^{-8} where as for the case of 15 samples, it is 7.64×10^{-6}, which is two orders of magnitude higher than for 100 samples. Fig. 3 also shows MSE for the two test data (30 samples) for both cases. It is seen that the MSE for both test data is of the order of two times the corresponding MSE values for 15 and 100 samples.

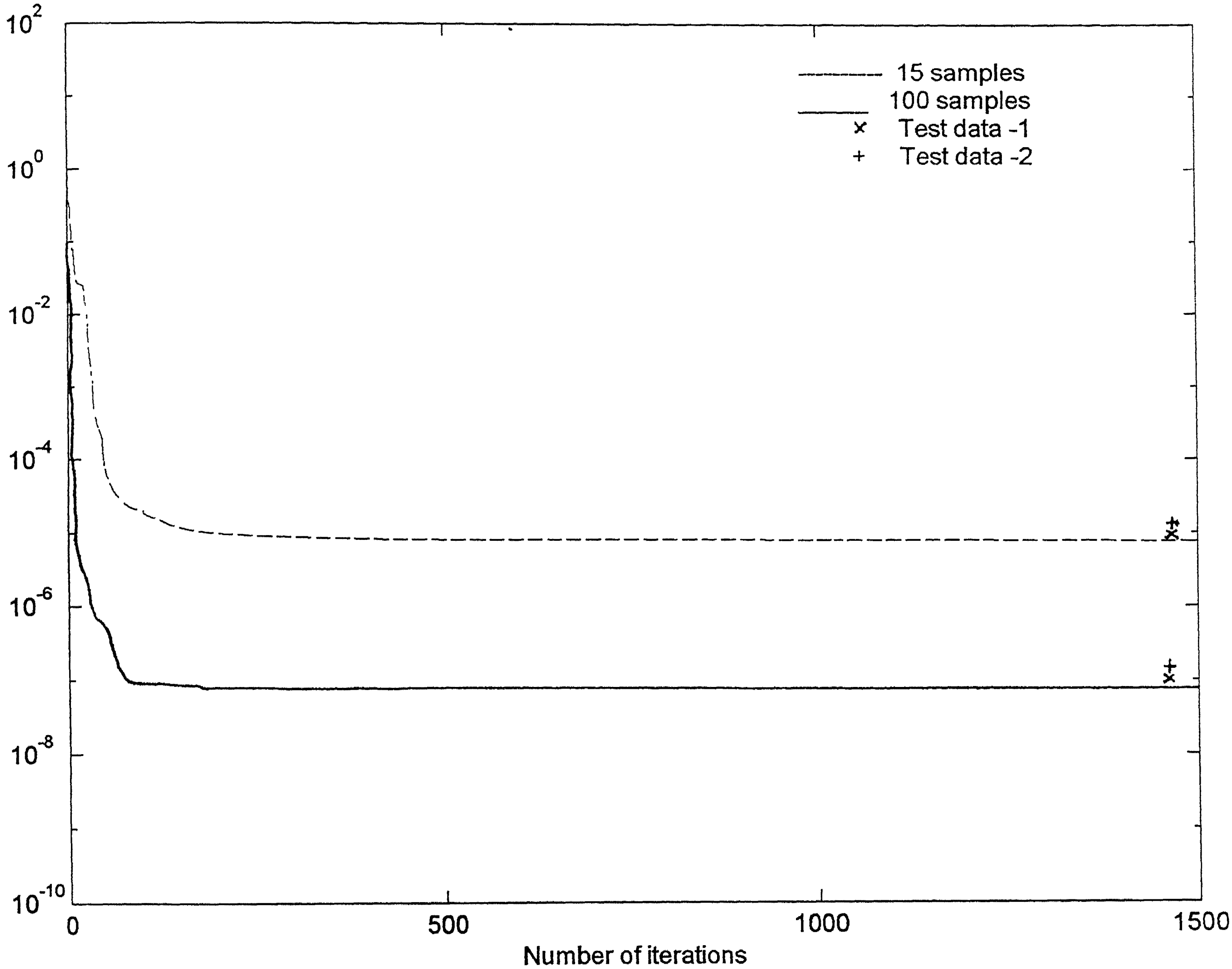


Fig. 3 Comparison of MSE for 15 and 100 training samples

Notwithstanding the superior training for the case of 100 samples, for the case of 15 samples, in absolute terms, the training level achieved was satisfactory, as also was the validation test. Hence, all the results presented are only for the case of 15 samples.

For randomly selected 15 samples of θ, BAT, BAD, V, PW, HT and W_z, the corresponding R, Ψ and T were calculated as explained in 4.1. In addition, two other sets of 30 samples each were similarly generated for use as test data. The network tuning parameters were varied till acceptable network training was achieved as per criteria set out in Appendix B. Now a set of 10 samples, randomly taken from the range table, is used to predict R, Ψ and T. The results are given in Table 1 and graphically compared with the actual values in Fig. 4. As may be seen, the predicted values compare well with the actual values.

4.4 Model 2

This model is the one, which would be of more use in real-life applications. A soldier would want to know the firing angle (θ) he should use, the bearing correction (Ψ) he should apply and the time of flight (T) for shell to reach the target. The time of flight is required to set the fuse such that the shell explodes after a fixed time – the time being chosen to be just prior to shell reaching the target and thereby ensuring that shell explodes in air, just before hitting the ground. The information made available prior to firing of the shell is the ambient atmospheric conditions: head/tail wind (W_x), crosswind (W_z), muzzle velocity (V), projectile weight (PW), atmospheric temperature (BAT) and density (BAD). Thus, for the desired range under the prevailing atmospheric conditions, we wish to develop a model that would predict the firing angle, bearing correction and time of flight. The schematic of such a model is shown in fig. 5.

Table 1. Model 1: Comparison of Actual and Predicted Range, Bearing correction (Ψ), Time of flight (T) for the given value of θ

θ in Mils	Range in m		Ψ in Mils		T in sec	
	Actual	Predicted	Actual	Predicted	Actual	Predicted
7.7	1000.8	1000	0.097	0.098	1.301	1.3
11.8	1498	1498.5	0.579	0.573	1.898	1.89
25.3	3019.3	3020	-0.071	-0.07	4.119	4.12
35.5	3981.6	3983	2.073	2.075	5.563	5.561
40.9	4447.8	4446	3.557	3.56	6.348	6.34
52.7	5424.6	5423	3.604	3.6	8.049	8.05
65.8	6406.9	6406	5.545	5.5	9.814	9.82
72.9	7053.1	7051	1.843	1.845	11.106	11.1
88.4	8045.9	8046.5	5.227	5.22	13.296	13.3
96.9	8195.3	8194.6	8.991	9	13.6	13.61

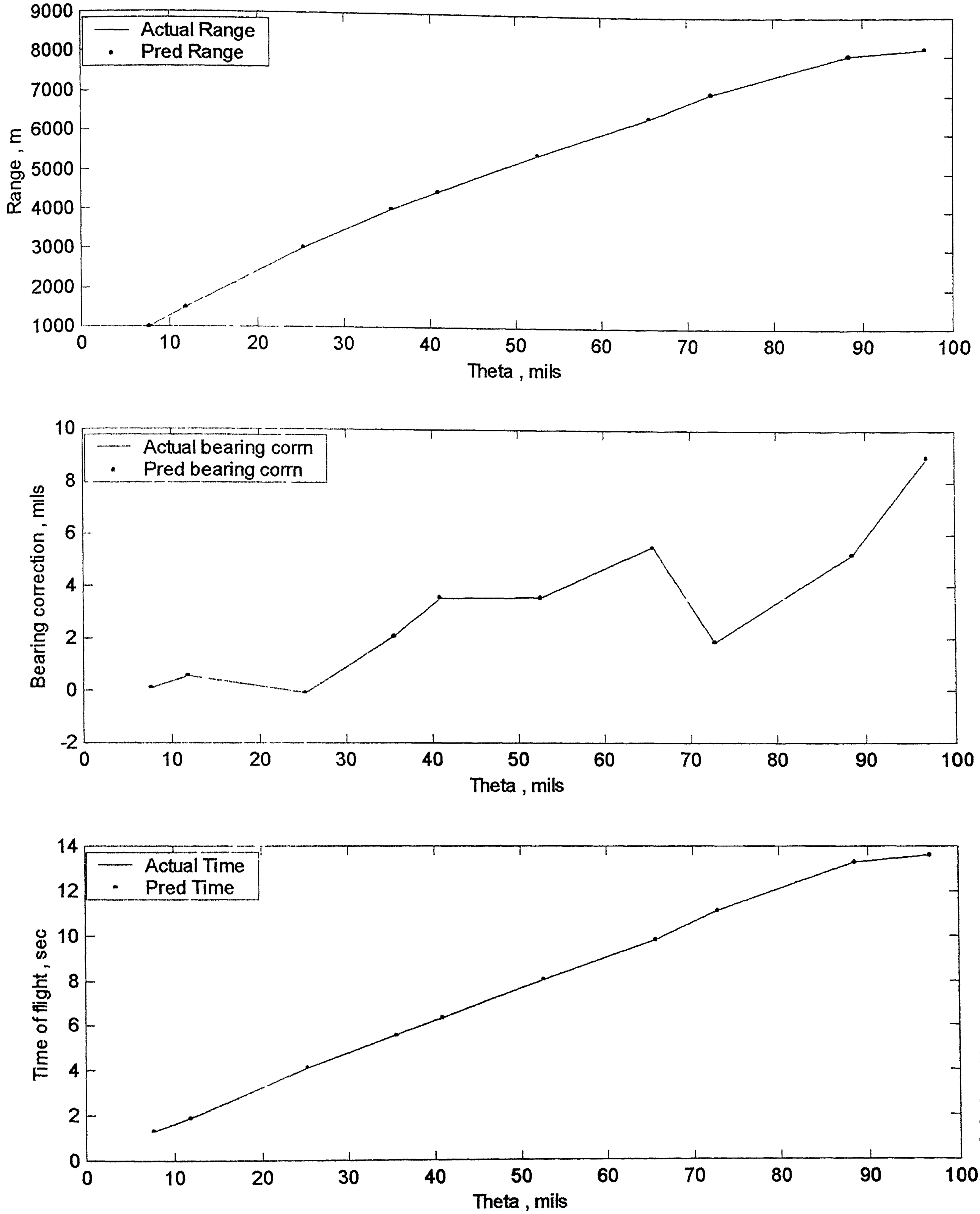


Fig. 4 Comparison of Actual and Predicted range, Bearing Correction and Time of Flight with Theta for Model 1

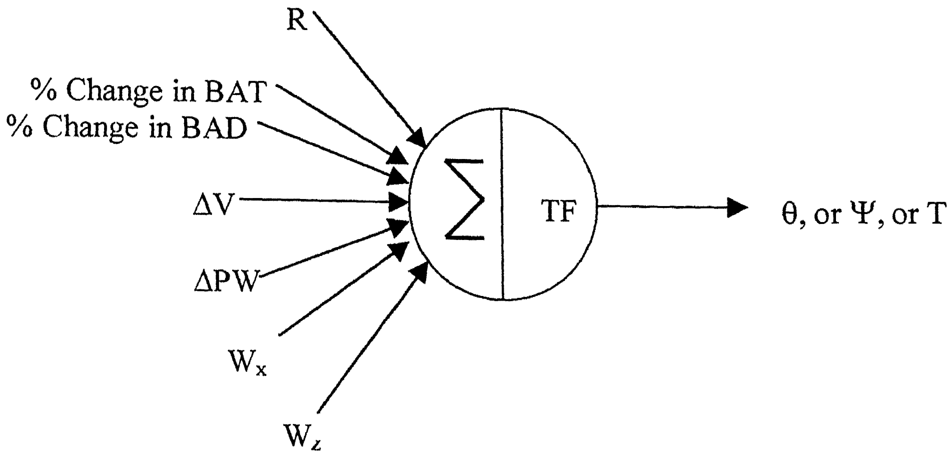


Fig.5 Schematic Representation of FFNN For model 2

Before attempting the above model, it was decided to first study simpler modelling problem. The number of inputs to the network were restricted to only two: the desired range and one from BAT, BAD, ΔV, ΔPW, W_x, W_z. Fig.6 shows schematic of six such submodels studied. It may be noted that, as in Model 1, the BAT and BAD are given in percent change from nominal values, and the ΔV, ΔPW are given in terms of difference from the corresponding nominal values.

The results for Model 2 are shown in Table 2 and Fig.7. The predicted θ, Ψ, T compare well with the actual values, the prediction being marginally better for the lower value of θ. One explanation for such observed behaviour is the fact that at lower end of θ, the range obtainable is more sensitive to variation in θ; typically, increase of 1 mil in θ results in range increasing by 134 m for a nominal value of $\theta = 0.7$ mil, but at nominal value of $\theta = 802.3$ mils, 1 mil increase in θ would result in range increasing by only by 3.2m. Notwithstanding this sensitivity of range on nominal value of θ, the predicted values by Model 2 are quite acceptable for the whole range of θ values.

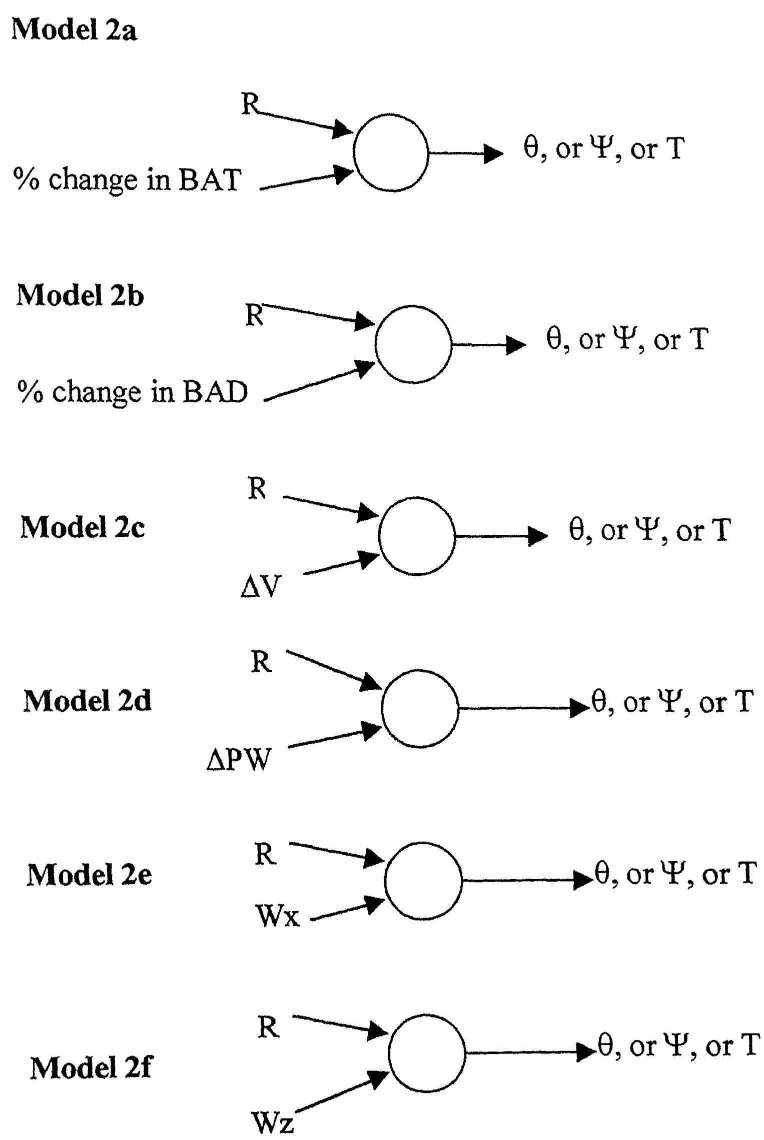


Fig.6 Schematic Representation of FFNN for sub models of Model 2

Table 2. Model 2: Comparison of Actual and Predicted Firing angle (θ), Bearing correction for Drift (Ψ) and Time of flight (T) for ambient atmospheric conditions, shell weight and muzzle velocity considerations.

Range in m	θ in Mils		Ψ in Mils		T in sec	
	Actual	Predicted	Actual	Predicted	Actual	Predicted
20100	509.10	508.85	-0.08	0.10	55.50	57.20
22800	738.60	744.50	38.60	37.75	74.90	74.80
23000	774.60	773.00	36.95	37.10	77.90	76.76
14000	233.50	234.00	4.10	4.88	30.30	31.20
10000	125.50	124.40	10.15	10.50	18.00	17.61
15100	272.50	273.30	15.04	15.50	34.20	34.29
12600	189.70	191.00	17.85	18.80	25.70	25.53
4100	36.50	36.40	0.0	0.10	5.80	5.60
1100	8.50	8.70	-0.10	-0.23	1.40	1.31
200	1.50	1.62	0.15	0.20	0.20	0.32

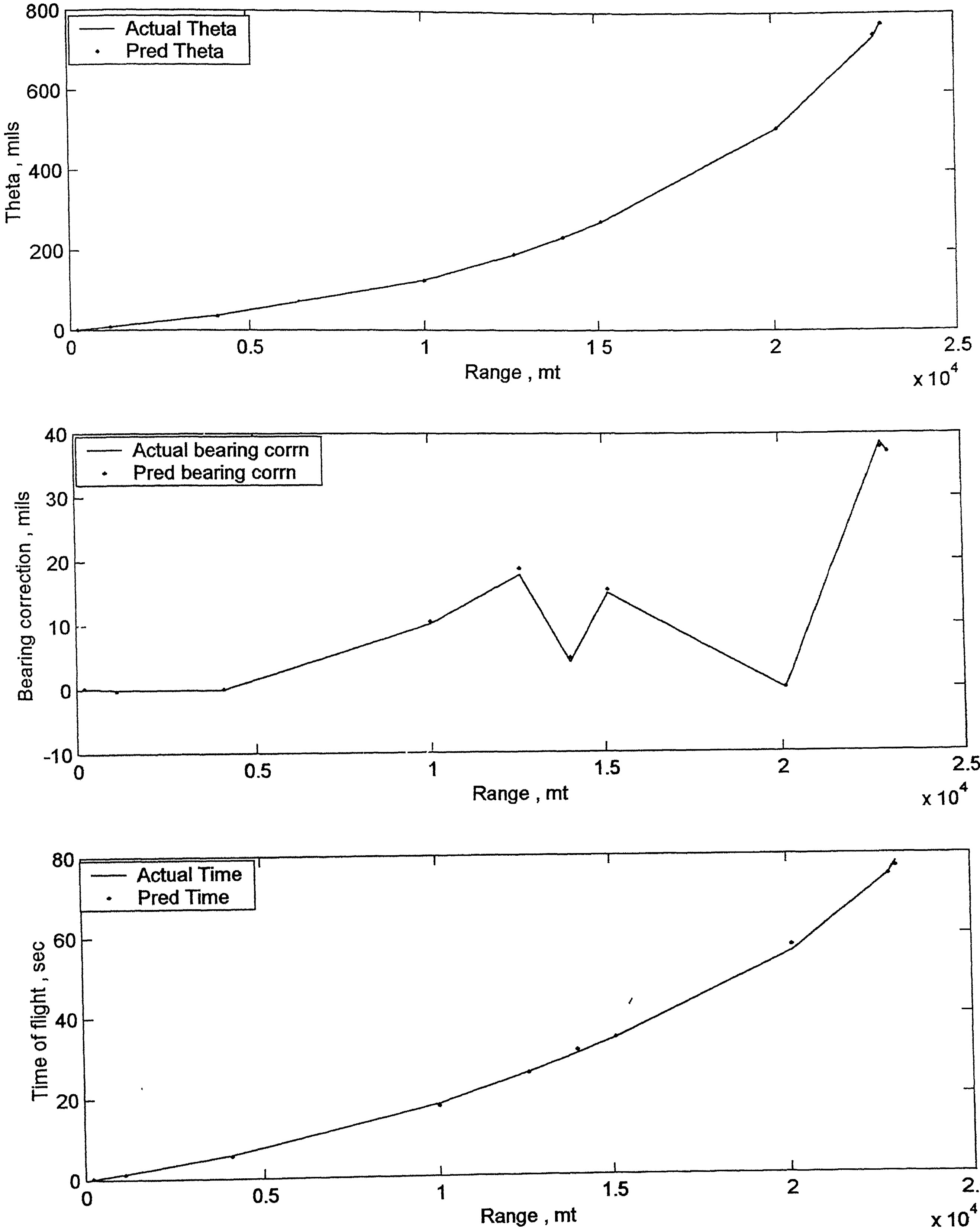


Fig. 7 Comparison of Actual and Predicted Theta, Bearing correction and Time of flight varying Range, V, PW, BAT, BAD, W_x and W_z for Model 2.

The results for the submodels Model 2a to Model 2f are discussed next. It was realized that all the submodels could be viewed as a special case of model 2. It would be, therefore, of interest to see how the results from the submodels compare with these from Model 2 by keeping R and only one of the input variables, corresponding to the submodel, nonzero and the rest of the input variables to zero. For example, for submodel Model 2a, the Model 2 would have R and % change in BAT as nonzero inputs, and the rest of these, i.e., % change in BAD, ΔV, ΔPW, W_x, and W_z equals to zero during the prediction phase. The results so obtainable via submodels and via Model 2 are compared in Table 3 - 8 and Figs. 8 - 13. The results as a special case of Model 2 corresponding to the submodels are given under the heading 'Special - Model 2'. It is noted that, as expected, the results via the submodels are uniformly better than those obtained as special case of Model 2.

4.5 Model 3

It is of interest to develop a model that can yield standard range (X) obtainable under standard conditions - temperatures and density corresponding to sea level values in standard atmosphere, no head/tail wind, shell having standard muzzle velocity and weight. The data assumed to be available for such modelling is the measured range (R) for existing ambient temperature, density, head/tail wind, muzzle velocity and shell weight. Two distinct approaches are used and the models, named model 3A and model 3B, have been developed for the purpose of predicting standard range.

Model 3A

The standard range differs from the non-standard range due to the effect of i) variations in atmospheric temperature and density, ii) amount of head/tail wind, iii)

Table 3. Model 2a: Comparison of Actual And Predicted Angle of Firing (θ), Bearing Correction for Drift (Ψ) and Time of Flight (T) for Ballistic air temperature (BAT) Consideration.

Range in m	θ in Mils			Ψ in Mils			T in sec		
	Actual	Model 2a	Special - Model 2	Actual	Model 2a	Special - Model 2	Actual	Model 2a	Special - Model 2
20100	509.10	509.10	510.20	17.20	17.20	17.50	55.50	55.50	57.20
22800	738.60	740.30	735.02	25.70	25.48	24.90	74.90	75.20	73.35
23000	774.60	773.00	765.57	27.60	28.20	26.60	77.90	78.00	77.37
14000	233.50	234.00	232.70	8.90	8.99	9.30	30.30	30.30	30.54
10000	125.50	126.01	124.01	5.20	5.17	5.28	18.00	18.00	18.15
15100	272.50	273.30	270.82	10.10	10.30	10.61	34.20	34.23	33.67
12600	189.70	189.04	190.50	7.60	7.60	7.90	25.70	25.60	26.30
4100	36.50	36.40	36.11	1.50	1.53	1.41	5.80	5.76	6.01
1100	8.50	8.70	8.80	0.30	0.30	0.35	1.40	1.40	1.45
200	1.50	1.53	1.62	0.10	0.12	0.12	0.20	0.21	0.25

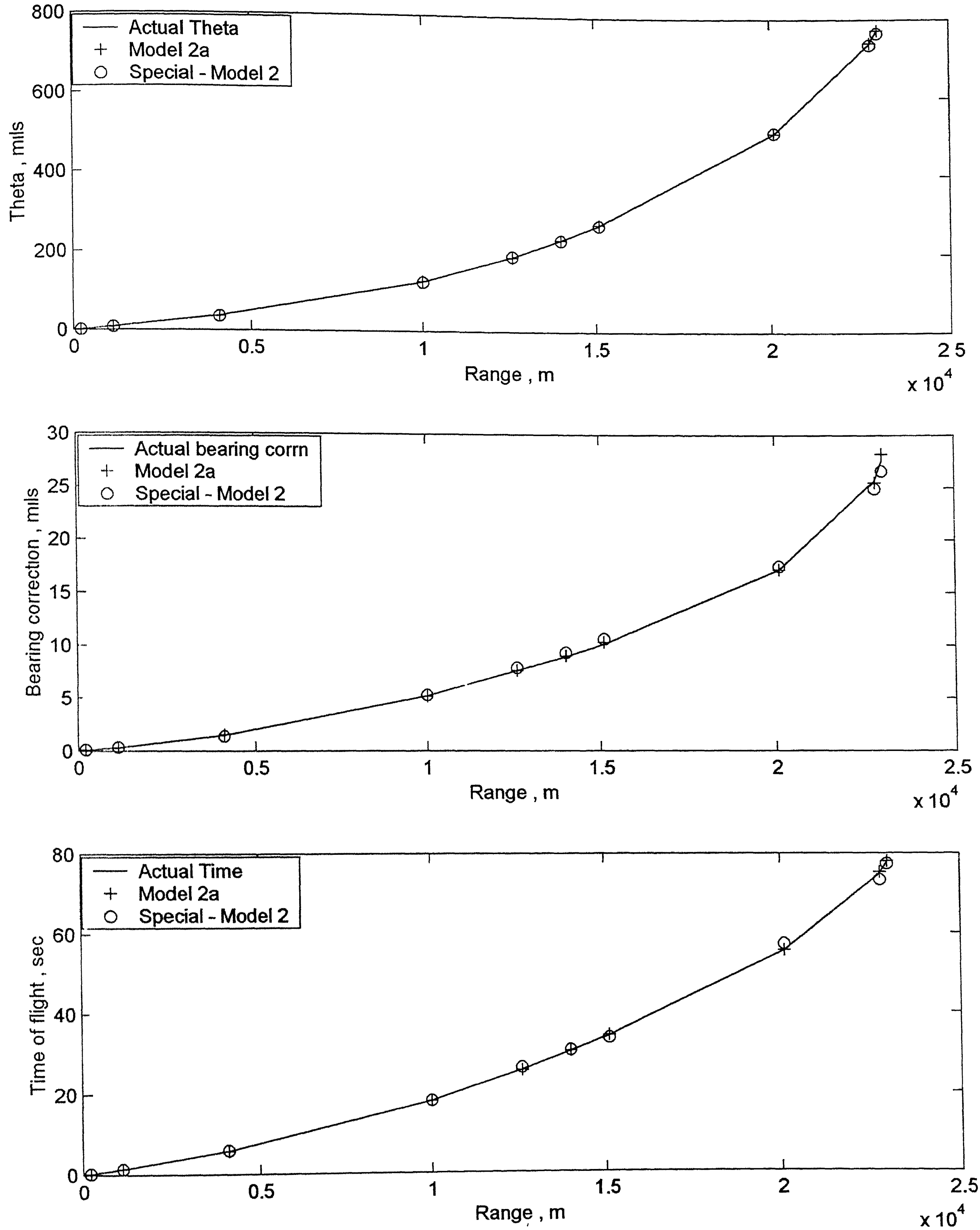


Fig. 8 Comparison of Actual and Predicted Theta, Bearing correction, and time of flight for varying Range and BAT.

Table 4. Model 2b: Comparison of Actual And Predicted Angle of Firing (θ), Bearing Correction for Drift (Ψ) and Time of Flight (T) for Ballistic air density Consideration.

Range in m	θ in Mils			Ψ in Mils			T in sec		
	Actual	Model 2b	Special - Model 2	Actual	Model 2b	Special - Model 2	Actual	Model 2b	Special - Model 2
20100	509.10	509.10	510.20	17.20	17.22	17.50	55.50	55.50	57.20
22800	738.60	741.30	735.02	25.70	25.40	24.90	74.90	75.00	73.35
23000	774.60	773.20	765.57	27.60	28.00	26.60	77.90	78.00	77.37
14000	233.50	233.60	232.70	8.90	8.91	9.30	30.30	30.30	30.54
10000	125.50	125.01	124.01	5.20	5.16	5.28	18.00	18.00	18.15
15100	272.50	273.43	270.82	10.10	10.30	10.61	34.20	34.23	33.67
12600	189.70	189.04	190.50	7.60	7.50	7.90	25.70	25.60	26.30
4100	36.50	36.45	36.11	1.50	1.53	1.41	5.80	5.76	6.01
1100	8.50	8.80	8.80	0.30	0.32	0.35	1.40	1.40	1.45
200	1.50	1.53	1.62	0.10	0.15	0.12	0.20	0.21	0.25

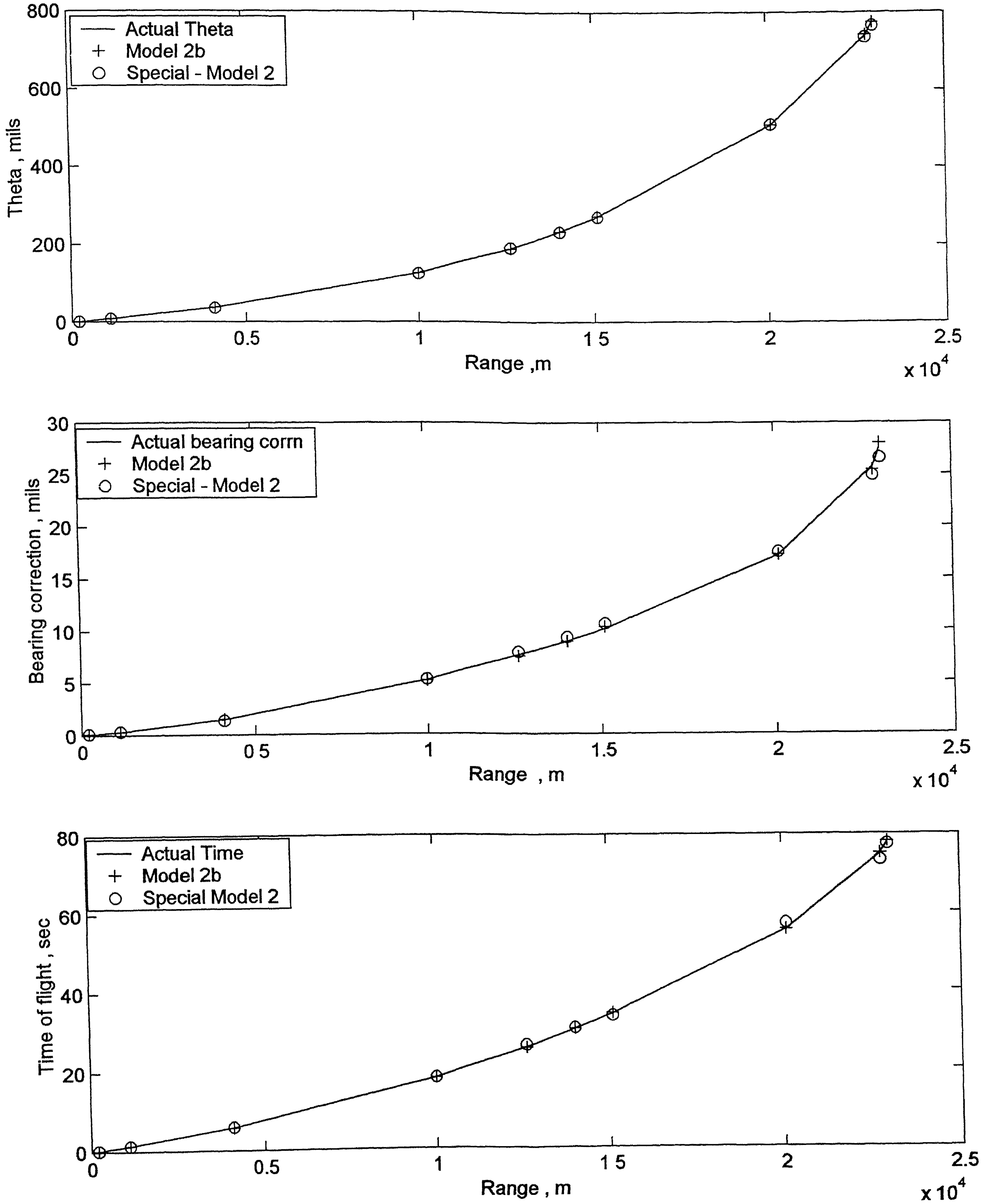


Fig. 9 Comparison of Actual and Predicted Theta, Bearing correction and Time of flight for varying range and BAD.

Table 5. Model 2c: Comparison of Actual And Predicted Angle of Firing (θ), Bearing Correction for Drift (Ψ) and Time of Flight (T) for Velocity Consideration.

Range in m	θ In Mils			Ψ In Mils			T in Sec		
	Actual	Model 2c	Special - Model 2	Actual	Model 2c	Special - Model 2	Actual	Model 2c	Special - Model 2
20100	509.10	509.10	510.20	17.20	17.26	18.10	55.50	55.46	60.20
22800	738.60	740.30	735.02	25.70	25.80	24.59	74.90	75.02	70.35
23000	774.60	773.00	765.57	27.60	28.50	25.80	77.90	78.20	77.37
14000	233.50	234.00	232.70	8.90	9.01	9.80	30.30	30.31	30.54
10000	125.50	126.01	124.01	5.20	4.97	5.28	18.00	18.30	18.15
15100	272.50	273.30	270.82	10.10	10.40	10.61	34.20	34.35	33.67
12600	189.70	189.04	190.50	7.60	7.38	7.90	25.70	25.70	26.30
4100	36.50	36.40	36.11	1.50	1.61	1.31	5.80	5.76	6.01
1100	8.50	8.70	8.80	0.30	0.35	0.40	1.40	1.37	1.45
200	1.50	1.53	1.62	0.10	0.15	0.20	0.20	0.22	0.25

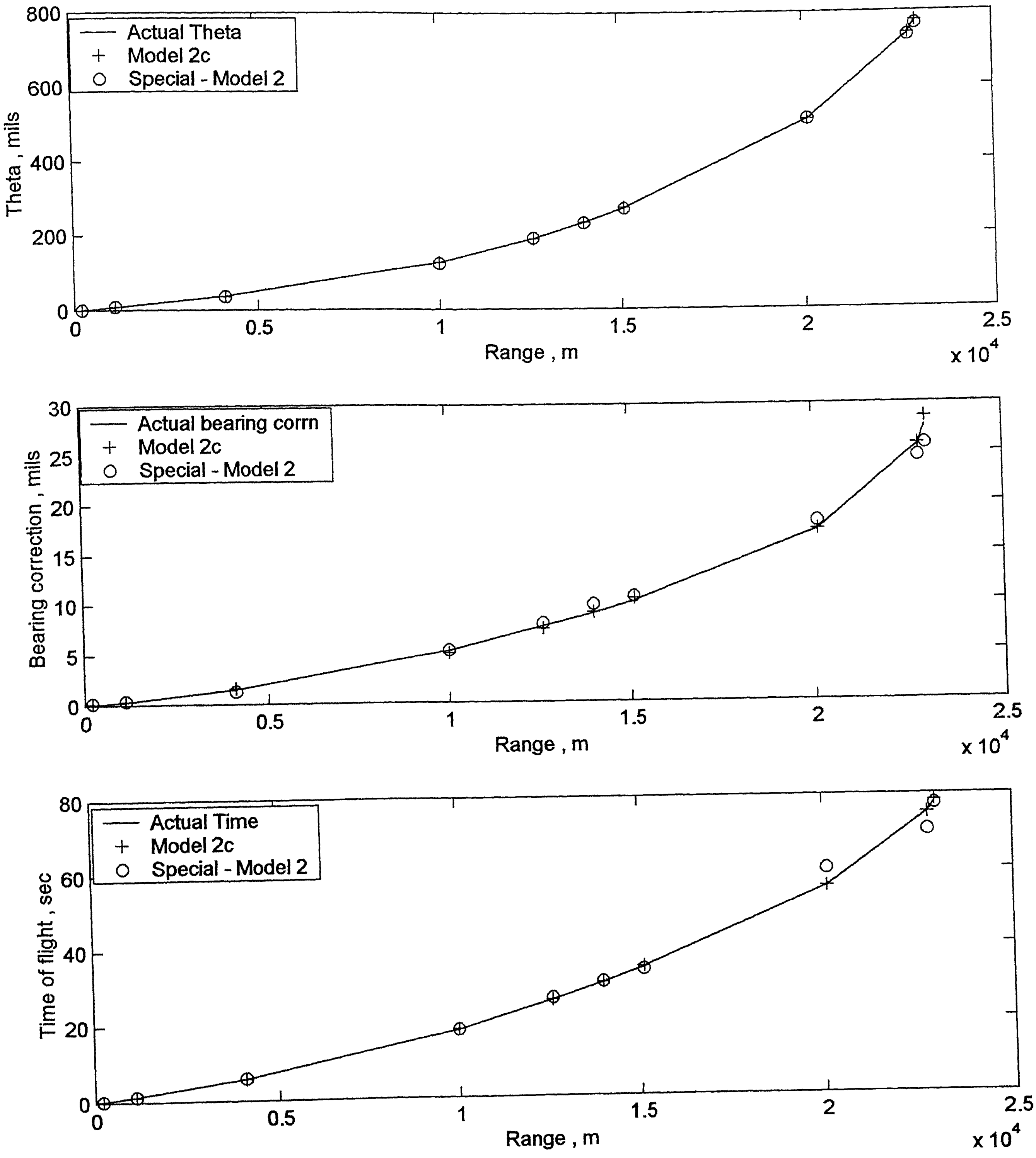


Fig. 10 Comparision of Actual and Predicted Theta, Bearing correction, and Time of flight for varying Range and V.

Table 6. Model 2d: Comparison of Actual And Predicted Angle of Firing (θ), Bearing Correction for Drift (Ψ) and Time of Flight (T) for Projectile weight Consideration.

Range in m	θ in Mils			ψ in Mils			T in sec		
	Actual	Model 2d	Special - Model 2	Actual	Model 2d	Special - Model 2	Actual	Model 2d	Special - Model 2
20100	509.10	509.12	510.20	17.20	17.25	17.50	55.50	55.50	57.20
22800	738.60	740.30	735.02	25.70	25.30	24.90	74.90	75.10	74.50
23000	774.60	773.25	765.57	27.60	28.00	26.60	77.90	78.00	77.37
14000	233.50	233.40	232.70	8.90	8.90	9.30	30.30	30.31	30.54
10000	125.50	125.60	124.01	5.20	5.16	5.28	18.00	18.10	18.15
15100	272.50	272.30	272.82	10.10	10.30	10.61	34.20	34.23	33.67
12600	189.70	189.80	190.50	7.60	7.70	7.90	25.70	25.60	26.30
4100	36.50	36.40	36.11	1.50	1.53	1.41	5.80	5.70	6.01
1100	8.50	8.80	8.40	0.30	0.32	0.35	1.40	1.40	1.45
200	1.50	1.52	1.60	0.10	0.15	0.12	0.20	0.21	0.25

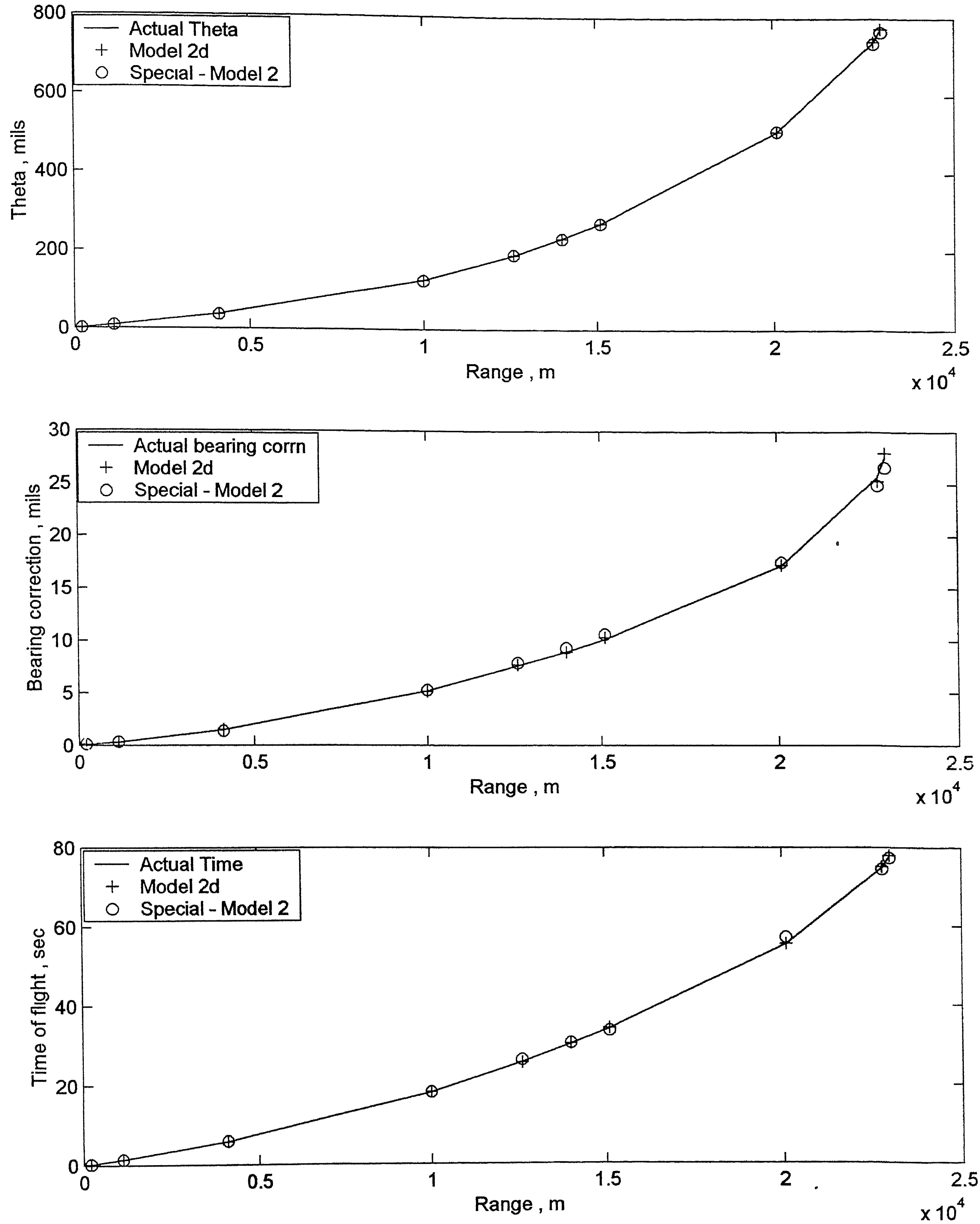


Fig. 11 Comparison of Actual and Predicted Theta, Bearing correction, and time of flight for varying Range and PW.

Table 7. Model 2e: Comparison of Actual and Predicted Angle of firing (θ), Bearing Correction for drift (Ψ) and time of flight (T) for head/tail wind consideration.

Range in m	θ in Mils			Ψ in Mils			T in sec		
	Actual	Model 2e	Special - Model 2	Actual	Model 2e	Special - Model 2	Actual	Model 2e	Special - Model 2
20100	509.10	510.10	510.20	17.20	17.26	18.10	55.50	55.50	56.20
22800	738.60	735.10	736.02	25.70	25.70	24.59	74.90	74.90	73.35
23000	774.60	774.00	765.00	27.60	27.70	25.80	77.90	77.90	77.37
14000	233.50	233.50	232.50	8.90	8.90	9.60	30.30	30.20	30.54
10000	125.50	125.50	124.00	5.20	5.15	5.20	18.00	18.08	18.15
15100	272.50	272.30	270.82	10.10	10.20	10.50	34.20	34.35	33.67
12600	189.70	189.40	190.50	7.60	7.40	7.60	25.70	25.67	26.30
4100	36.50	36.54	36.10	1.50	1.41	1.41	5.80	5.76	6.01
1100	8.50	8.47	8.70	0.30	0.35	0.40	1.40	1.38	1.45
200	1.50	1.53	1.60	0.10	0.15	0.18	0.20	0.19	0.25

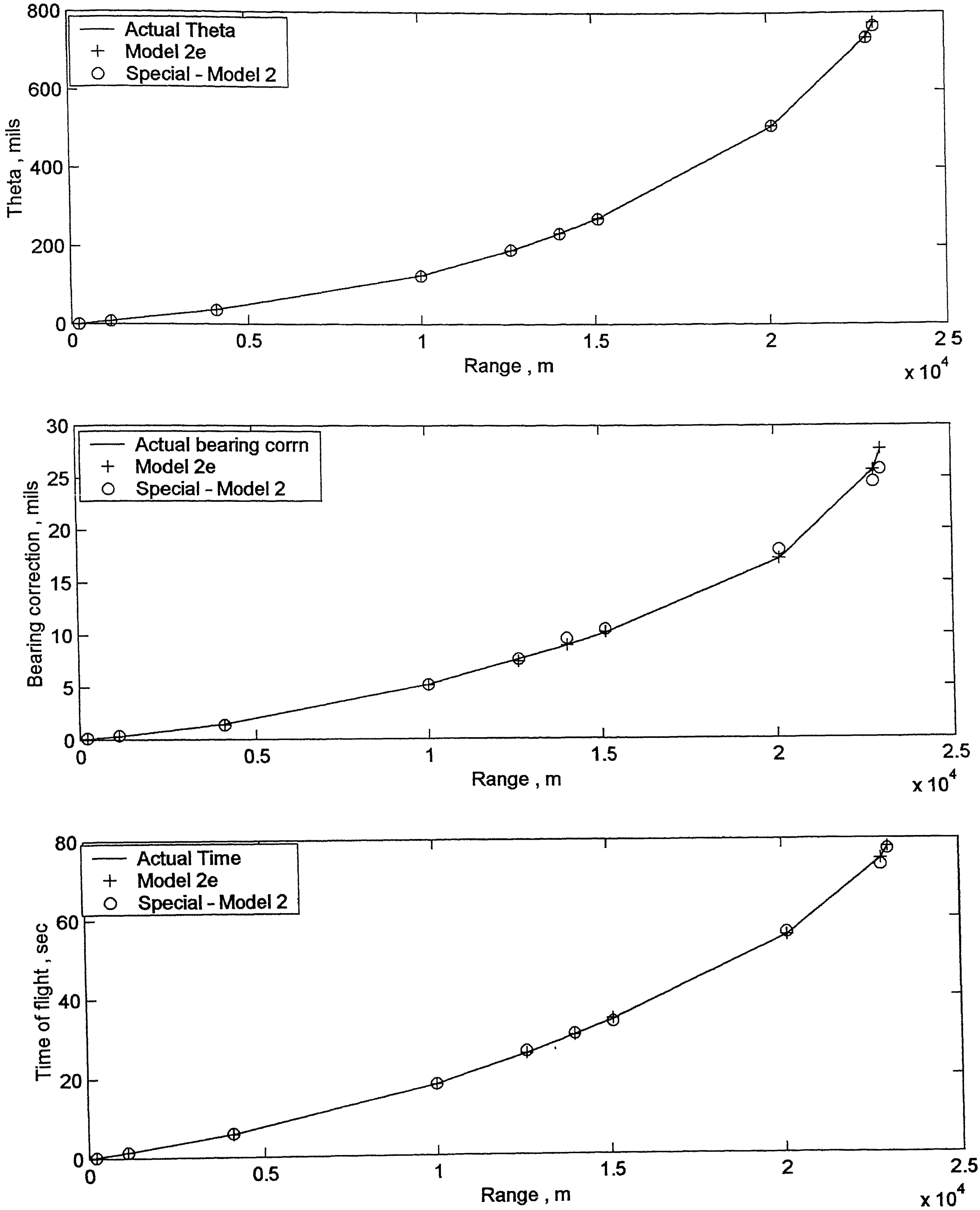


Fig. 12 Comparison of the Actual and Predicted Theta, Bearing correction, and Time of flight for varying Range and W_x.

Table 8. Model 2f: Comparison of Actual and Predicted Angle of Firing (θ), Bearing Correction for Drift (Ψ) and Time of Flight (T) for crosswind consideration.

Range in m	θ in Mils			Ψ in Mils			T in sec		
	Actual	Model 2f	Special - Model 2	Actual	Model 2f	Special - Model 2	Actual	Model 2f	Special - Model 2
20100	509.10	509.00	509.20	0.08	0.08	0.07	55.50	55.56	56.20
22800	738.60	737.10	736.52	38.60	38.75	38.20	74.90	74.49	73.35
23000	774.60	774.00	770.00	36.95	37.07	37.17	77.90	76.90	77.37
14000	233.50	233.50	232.50	4.10	4.20	4.22	30.30	30.32	30.54
10000	125.50	125.50	125.00	10.15	10.20	10.32	18.00	18.18	18.15
15100	272.50	272.30	271.82	15.04	15.05	15.40	34.20	34.05	33.67
12600	189.70	189.40	190.10	17.85	17.75	17.70	25.70	25.70	26.30
4100	36.50	36.54	36.40	0.0	0.01	0.01	5.80	5.46	6.01
1100	8.50	8.45	8.60	0.10	0.15	0.17	1.40	1.48	1.45
200	1.50	1.53	1.56	0.15	0.13	0.15	0.20	0.20	0.25

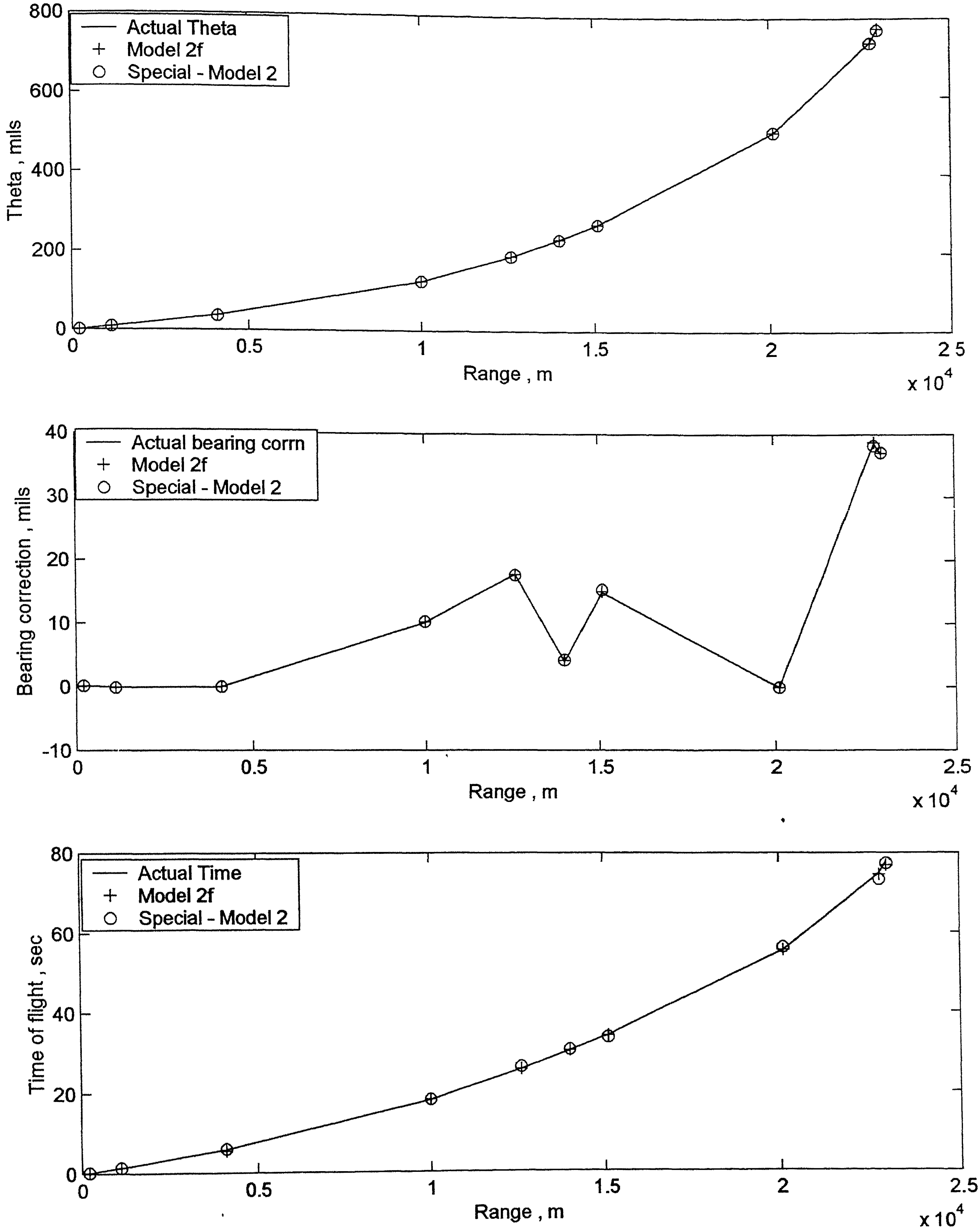


Fig. 13 Comparison of Actual and Predicted Theta, Bearing correction and Time of flight for varying Range and W_z.

variations in muzzle velocity and shell weight from the nominal values. If one could evaluate sensitivity coefficients that estimate how much the range varies per unit change in each of these, then one can calculate the standard from the non-standard range as follows:

$$X = R - \left(\frac{\partial R}{\partial T}\right)\Delta T - \left(\frac{\partial R}{\partial \rho}\right)\Delta\rho - \left(\frac{\partial R}{\partial W_X}\right)\Delta W_x - \left(\frac{\partial R}{\partial V}\right)\Delta V - \left(\frac{\partial R}{\partial PW}\right)\Delta PW \qquad (4.2)$$

where X is the standard range, R the non-standard range, $\left(\frac{\partial R}{\partial T}\right)$, $\left(\frac{\partial R}{\partial \rho}\right)$, $\left(\frac{\partial R}{\partial W_X}\right)$, $\left(\frac{\partial R}{\partial V}\right)$ and $\left(\frac{\partial R}{\partial PW}\right)$ are, respectively, the sensitivity coefficients with respect to temperature, density, head/tail wind, muzzle velocity and shell weight. $\Delta T = T - T_o$, $\Delta\rho = \rho - \rho_o$, $\Delta V = V - V_o$, $\Delta PW = PW - PW_o$, where T, ρ, V, PW are the values of temperature, density, muzzle velocity and shell weight at time of firing, and T_o, ρ_o are the sea level standard values, V_o and PW_o are the nominal value of muzzle velocity and shell weight. The W_x is the value of head or tail wind (W_x is positive for tail wind and negative for head wind). To estimate sensitivity coefficients, five separate neural models (shown in fig.14) were developed. The methodology used is based on the Delta method proposed in ref. [8,13] for estimating aircraft stability and control derivatives from the flight data. The derivatives (sensitivity coefficients $\left(\frac{\partial R}{\partial T}\right)$, $\left(\frac{\partial R}{\partial \rho}\right)$, etc.) can also be estimated by using the Delta method. Here the FFNN is trained to map one of the input variables from T, ρ, W_x, V, PW to the output variable range(R). The network input is now given a small perturbation in both increasing and decreasing directions. Such a modified input file is now presented to the trained network to predict the Range, R at its output node. The

difference in the predicted value of the range from the value predicted for the original value of the input is attributed to the perturbation of the chosen input variable. The difference so calculated in the value of range divided by the perturbation value introduced in the input variable yields the corresponding sensitivity coefficients. For example, to estimate $\left(\frac{\partial R}{\partial T}\right)$, the temperature T is varied to T + ΔT and T - ΔT and the corresponding predicted range R^+ and R^- are recorded. Then

$\left(\frac{\partial R}{\partial T}\right) = \frac{R^+ - R^-}{2\Delta T}$. For the purpose of illustration, non-standard range corresponding to standard range of 20000 m was calculated for various combinations of T, ρ, W_x, V and PW. A set of 15 such data was used to train the five neural networks having non-standard range as the output and one out of T, ρ, W_x, V, PW as the input. The trained network was used to estimate the five required sensitivity coefficients; one from each of the networks shown in fig.14. Finally, the standard range is calculated by using Eq. 4.2. A comparison of predicted and actual standard range is given in Table 9. Table 9 lists the non-standard range along with the sensitivity coefficients. Table 9 shows that the standard range estimates via sensitivity coefficients compare well with the actual standard range. However, this approach is a round about way of predicting standard range, and, therefore, a more direct method would be highly desirable. This search for a direct method led to the following model.

Model 3B

Model 3B has two versions, named Model 3B-1 and Model 3B-2 that were studied. Model 3B-1 is identical to Model 1, but it is used in a special way for estimating standard range. Like Model 1, output of the network is range R, which is mapped to θ,

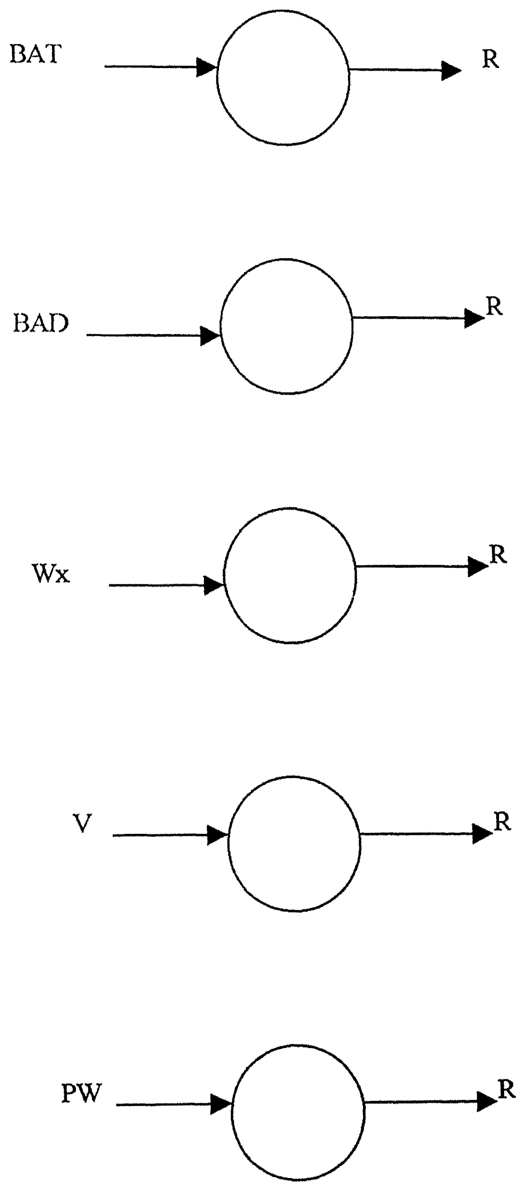


Fig.14 Schematic of Neural Models for Estimating Sensitivity Coefficients

Table 9. Model 3A: Comparison between actual and calculated range at standard atmospheric conditions.

R in m	$\frac{\partial R}{\partial V}$	$\frac{\partial R}{\partial W_X}$	$\frac{\partial R}{\partial T}$	$\frac{\partial R}{\partial \rho}$	$\frac{\partial R}{\partial PW}$	Actual X In m	Calculated X in m	Diff. Bet. Actual & Calculated X
19605.1	27.89	20.80	-0.31	-106.86	45.00	20000.0	20011.0	-11.00
20522.3	27.90	20.80	0.23	-109.08	45.01	20000.0	20000.9	-0.94
20319.3	27.90	20.80	-1.09	-107.19	42.04	20000.0	20000.6	-0.60
20336.4	27.91	20.80	0.15	-109.08	45.00	20000.0	20004.5	-4.59
20430.0	27.90	20.80	-0.92	-107.42	45.01	20000.0	20004.1	-4.10
19613.1	27.87	20.80	-1.04	-107.15	45.00	20000.0	20006.6	-6.61
20264.8	27.90	20.79	0.19	-109.10	44.97	20000.0	20001.9	-1.98
19966.1	27.91	20.80	-0.13	-108.44	44.99	20000.0	20002.4	-2.47
19436.4	27.90	20.80	-1.39	-103.30	44.99	20000.0	19993.2	1.72
19776.4	27.90	20.64	0.26	-109.20	42.53	20000.0	19998.6	1.39
20224.1	27.90	20.80	0.19	-109.10	45.01	20000.0	20002.5	-2.58
19324.0	27.90	20.75	-0.34	-107.71	42.54	20000.0	20014.9	-14.90
20593.9	27.86	20.80	0.26	-109.18	43.88	20000.0	20002.7	-2.72
19878.9	27.90	20.79	-0.75	-107.77	45.00	20000.0	20004.1	-4.11
19945.6	27.90	20.69	0.20	-109.10	44.96	20000.0	20001.3	-1.34

ΔV, %BAT, %BAD, ΔPW and Head/tail wind (W_x). However, mapping of the inputs to bearing correction Ψ and time of flight T is not required as done for Model 1. The network is trained on randomly selected 40 data taken from the range table. Now, the trained network is used to predict standard range for each of the firing angle θ by setting $\Delta V = \Delta PW = \%BAT = \%BAD = W_x = 0$. The standard range for various value of θ so obtained is given in table 10.

Model 3B-2 is similar to the Model 3B-1 except that instead of ΔV and ΔPW, percent change from the nominal value of the muzzle velocity and the shell weight were used as the input variables. The trained network was again used to predict standard range for various firing angles by putting all variables except θ to zero. The results are shown in Table 10.

In Table 10 and Fig.15, the predicted standard R via Model 3B-1 and Model 3B-2 are compared with that from the range table. Though both the models yield standard R which compares well with that given by the range table, it is observed that the Model 3B-2 gives relatively better results compared to Model 3B-1. Also, it may be noted that predictions for standard range are relatively better for lower values of θ, i.e., there is slight decrease in prediction accuracy of standard R as θ increases.

It is emphasized that both options of Model 3B are more suitable and straightforward to use for real life applications. In contrast, model 3A requires multiple networks to be used and calculation of the standard range using results from these networks.

Table 10. Comparison of Actual Range at standard condition for Different types of model.

θ in degree	R-non standard in m	X-standard in m	X- Model 3B-1 in m	X- Model 3B-2 in m
25.41	18922.00	19100.00	19107.00	19101.00
25.72	19315.00	19200.00	19205.00	19199.00
26.03	18921.00	19300.00	19304.00	19298.00
26.34	19439.00	19400.00	19404.00	19399.00
26.66	19688.00	19500.00	19503.00	19498.00
26.98	19097.00	19600.00	19602.00	19598.00
27.30	20226.00	19700.00	19701.00	19696.00
27.62	19900.00	19800.00	19800.00	19796.00
27.96	19442.00	19900.00	19900.00	19896.00
28.29	19996.00	20000.00	20001.00	19997.00
28.64	19861.00	20100.00	20101.00	20097.00
28.98	19991.00	20200.00	20201.00	20197.00
29.33	20545.00	20300.00	20302.00	20299.00
29.69	20667.00	20400.00	20402.00	20399.00
30.05	19922.00	20500.00	20502.00	20499.00
30.42	20355.00	20600.00	20603.00	20600.00
30.79	20748.00	20700.00	20703.00	20700.00
31.17	20493.00	20800.00	20804.00	20801.00
31.56	21139.00	20900.00	20903.00	20900.00
31.95	20933.00	21000.00	21003.00	21001.00
32.36	21505.00	21100.00	21104.00	21102.00
32.77	20819.00	21200.00	21204.00	21202.00
33.18	21467.00	21300.00	21303.00	21301.00
33.62	21531.00	21400.00	21403.00	21402.00
34.05	21678.00	21500.00	21503.00	21502.00
34.50	21394.00	21600.00	21602.00	21602.00
34.97	21627.00	21700.00	21701.00	21701.00
35.44	21738.00	21800.00	21800.00	21800.00
35.93	21625.00	21900.00	21898.00	21898.00
36.44	22254.00	22000.00	21997.00	21998.00
36.96	22141.00	22100.00	22095.00	22096.00
37.50	22003.00	22200.00	22192.00	22193.00
38.07	22370.00	22300.00	22291.00	22292.00
38.67	23179.00	22400.00	22390.00	22391.00
39.30	22571.00	22500.00	22488.00	22490.00
39.99	22071.00	22600.00	22590.00	22591.00
40.73	22452.00	22700.00	22692.00	22693.00
41.55	23353.00	22800.00	22796.00	22797.00
42.46	23171.00	22900.00	22903.00	22902.00
43.59	22827.00	23000.00	23021.00	23017.00

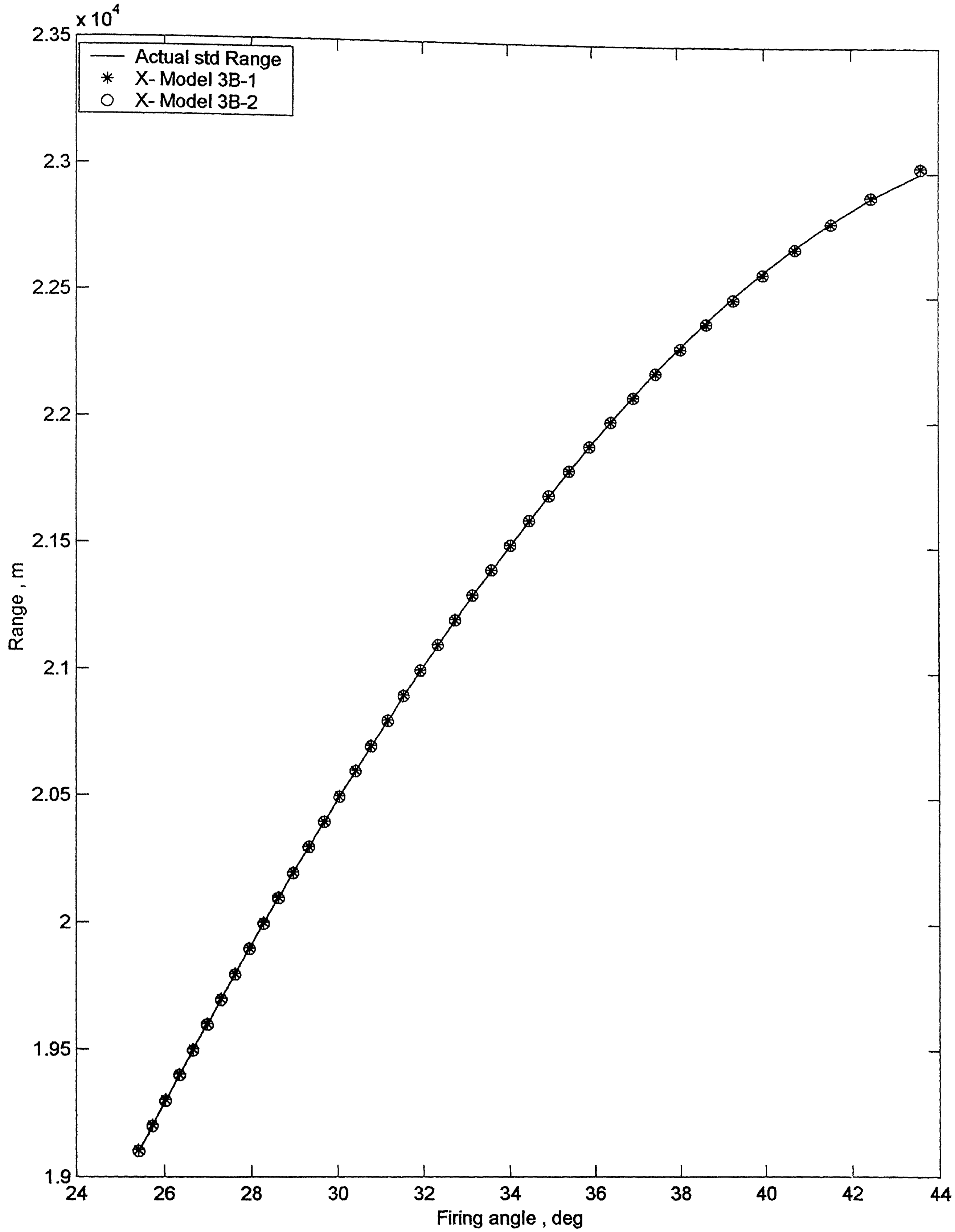


Fig. 15 Comparison of Actual and Predicted Standard Range for different types of model

CHAPTER 5

CONCLUSION

5.1 Conclusions

The validity of the neural modelling is demonstrated for three different applications relevant for artillery shells. A set of available firing data in terms of range, firing angle, shell weight, muzzle velocity, existing atmospheric conditions like temperature, density, head/tail wind, cross wind, time of flight, bearing correction are required to train the network with suitable input-output samples: which of these measured variables would form the inputs and the outputs is decided by the purpose of the neural model. In particular, for three distinct applications, a neural model has been identified and validated for B-Shell data. The results for all the models compare well with the known results. The strength of the neural models lies in the fact that once the network is trained, it can be used for on-line applications on the field of action – to predict the range for chosen angle of firing or to predict the firing angle for desired range, under the prevailing atmospheric conditions and for known shell weight and muzzle velocity.

The conventional approach, i.e., the mathematical models such as in-vacuo trajectory model, modified point mass model, six-degree-of-freedom model require knowledge of all the forces and moments acting on the shell. Evaluation of forces and moments, in turn, require aerodynamic coefficients as inputs and this fact limits the accuracy of predictions because the reliability of available estimates of these coefficients is not always high. In contrast, the proposed neural models do not require any mathematical model or its solution. This implies that the neural models do not require estimates of aerodynamic coefficients. Furthermore, if the neural network is trained on

the real data, it will automatically account for the initial conditions in an implicit way. But the mathematical models have no provisions to account for initial conditions such as gun jump and throw-off.

5.2 Suggestions for Future work

1) The training of the neural network depends on the architecture of the FFNNs, the activation function used by the neurons and the values selected for the tuning parameters. From the neural network point of view, lot of scope exists to explore and experiment with new evolving schemes and methodologies for improving the input-output mapping.

2) The present work has used the firing tables provided by ARDE, Pune. An attempt should be made to seek real firing data and validate the present models for such data. Using real data, it will be of interest to predict range for known firing angle, or to predict firing angle required for desired range using the mathematical models like the modified point mass model or six-degree-of-freedom model and also from the proposed neural Model 1 or Model 2. A comparison of results from mathematical models and neural models would show relative reliability of the predicted value for real life applications.

3) The data set supplied by ARDE, Pune is valid for weighted value of the head/tail wind and cross wind. This is an approximation to varying wind conditions at different altitudes through which the shell passes. It may be worthwhile to search for a way to account for varying wind conditions. The same comment also applies for finding a way to account for varying temperature and density conditions.

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Appendix A

Table F(i)

CHARGE 9 - SHELL 77B

Basic Data

Altitude Sea Level

1	2	3	4	5	6	7	8	9	10
Range	Firing Table elevation	Fuze setting	Correction to Fuze Setting to change Height of Burst by Down 10M	Effect on Range for increase of 1 mil in Firing Table elevation	Fork	Time of flight	Correction to Bearing for drift	Correction to Bearing for 1 knot cross wind	Correction to "Correction" to change Height of Burst by Down 10M
(X)	(A_E)	(FS)	$\left(\frac{\Delta_C FS}{-10MY_b}\right)$	$\left(\frac{\Delta_{EF} X}{+1m A_E}\right)$	(f)	(t)	($\Delta_C A_d$)	$\left(\frac{\Delta_C A_B}{1KT W_T}\right)$	
M	m			M	m	S	mL	m	points
100	.7			134.4	1	.1	.0	.00	
200	1.5			133.6	1	.2	.1	.00	
300	2.2			132.5	1	.4	.1	.01	
400	3.0			131.3	1	.5	.1	.01	
500	3.8			129.8	1	.6	.1	.01	
600	4.5			129.0	1	.7	.2	.01	
700	5.3			127.8	1	.9	.2	.01	
800	6.1			126.9	1	1.0	.2	.02	
900	6.9			125.5	1	1.1	.3	.02	
1000	7.7			124.4	1	1.3	.3	.02	
1100	8.5			123.3	1	1.4	.3	.02	
1200	9.3			122.3	1	1.5	.4	.03	
1300	10.1			121.2	1	1.7	.4	.03	
1400	11.0			120.1	1	1.8	.4	.03	
1500	11.8			119.0	1	1.9	.5	.03	
1600	12.6			117.8	1	2.1	.5	.03	
1700	13.5			116.9	1	2.2	.5	.04	
1800	14.4			115.8	1	2.3	.6	.04	
1900	15.2			114.8	1	2.5	.6	.04	
2000	16.1			113.6	1	2.6	.6	.04	
2100	17.0			112.6	1	2.8	.7	.05	
2200	17.9			111.7	1	2.9	.7	.05	
2300	18.8			110.5	1	3.0	.8	.05	
2400	19.7			109.5	1	3.2	.8	.05	
2500	20.6			108.5	1	3.3	.8	.06	
2600	21.5			107.5	1	3.5	.9	.06	
2700	22.5			106.4	1	3.6	.9	.06	
2800	23.4			105.6	1	3.8	.9	.06	
2900	24.3			104.4	1	3.9	1.0	.07	
3000	25.3			103.4	1	4.1	1.0	.07	
3100	26.3			102.6	1	4.2	1.1	.07	
3200	27.3			101.5	1	4.4	1.1	.07	
3300	28.3			100.6	1	4.5	1.1	.08	
3400	29.3			99.6	1	4.7	1.2	.08	
3500	30.3			98.7	1	4.8	1.2	.08	
3600	31.3			97.7	1	5.0	1.3	.08	
3700	32.3			96.7	1	5.1	1.3	.09	
3800	33.3			95.8	1	5.3	1.4	.09	
3900	34.4			94.8	1	5.4	1.4	.09	
4000	35.5			[illegible]					

Table F(ii)

CHARGE 9 - SHELL 77B

Corrections to range for non-standard conditions $\Delta_C X$

Altitude Sea Level

1	2	3	4	5	6	7	8	9	10	11
Range	1 M/S Muzzle Velocity (1 M/S V_O)		1 Knot Ballistic Wind (1KT W_B)		1% Ballistic Air Temperature (1% T_B)		1% Ballistic Density or Drag Coefficient (1% D_B)		1 [illegible] Projectile Weight	
(X)	Decrease (-)	Increase (+)	Head (W)	Tail (W)	Decrease (-)	Increase (+)	Decrease (-)	Increase +	Decrease (-)	Increase (+)
M	M	M	M	M	M	M	M	M	M	M
100	.3	-.2	0	-.0	-.0	+0	-.0	[illegible]	0	0
200	.5	-.5	.0	-.0	-.0	+0	-.0	[illegible]	-1	[illegible]
300	.8	-.7	.0	-.0	-.0	+.0	-.0	[illegible]	[illegible]	[illegible]
400	1.0	-1.0	0	-.0	-.0	+0	-.1	.1	[illegible]	[illegible]
500	1.2	-1.2	0	-.0	-.0	+0	-.1	.1	[illegible]	[illegible]
600	1.4	-1.5	0	-.0	-.0	+.0	-.2	.2	[illegible]	[illegible]
700	1.7	-1.7	0	-.0	-.0	+0	-.2	.2	-4	[illegible]
800	1.9	-1.9	.0	-.0	-.1	+.1	-.3	.3	-4	[illegible]
900	2.1	-2.2	.0	-.0	-.1	+.1	-.3	.3	-5	[illegible]
1000	2.4	-2.4	0	-.0	-.1	+.1	-.4	.4	-5	5
1100	2.6	-2.6	0	-.0	-.1	+.1	[illegible]	[illegible]	-6	5
1200	2.8	-2.9	.1	-.1	-.1	+.1	[illegible]	[illegible]	-6	6
1300	3.1	-3.1	.1	-.1	-.2	+.2	[illegible]	[illegible]	-6	6
1400	3.3	-3.3	.1	-.1	-.2	+.2	[illegible]	[illegible]	-7	7
1500	3.5	-3.5	.1	-.1	-.2	+2	-1.0	[illegible]	-7	7
1600	3.7	-3.8	.1	-.1	-.2	+.2	[illegible]	[illegible]	-8	8
1700	3.9	-4.0	.1	-.1	-.3	-.3	[illegible]	[illegible]	-8	8
1800	4.2	-4.2	.1	-.1	-.3	+.3	[illegible]	[illegible]	-9	9
1900	4.4	-4.4	.1	-.1	-.3	+.3	[illegible]	[illegible]	-9	9
2000	4.6	-4.6	2	-.1	-.4	+.4	[illegible]	[illegible]	-9	9
2100	4.8	-4.8	2	-.2	-.4	+.4	-1.9	[illegible]	-10	10
2200	5.0	-5.1	2	-.2	-.4	+.4	-2.0	[illegible]	-10	10
2300	5.2	-5.3	.2	-.2	-.5	+.5	[illegible]	[illegible]	-10	10
2400	5.4	-5.5	2	-.2	-.5	+.5	[illegible]	[illegible]	-11	10
2500	5.6	-5.7	2	-.2	-.6	+.6	-2.6	[illegible]	-11	11
2600	5.8	-5.9	.3	-.3	-.6	+.6	[illegible]	[illegible]	-11	11
2700	6.0	-6.1	.3	-.3	-.7	+.7	-3.1	[illegible]	-11	11
2800	6.2	-6.3	.3	-.3	-.7	+.7	-3.3	3.3	-12	12
2900	6.4	-6.5	.3	-.3	-.8	+.8	-3.5	3.5	-12	12
3000	6.6	-6.7	.3	-.3	-.8	+.8	-3.8	3.8	-12	12
3100	6.8	-6.9	.4	-.4	-.9	+.9	-4.0	4.0	-12	12
3200	7.0	-7.1	.4	-.4	-1.0	+1.0	-4.3	4.3	-13	13
3300	7.2	-7.3	.4	-.4	-1.0	+1.0	-4.6	4.6	-13	13
3400	7.4	-7.5	.4	-.4	-1.1	+1.1	-4.9	4.8	-13	13
3500	7.6	-7.7	.5	-.5	-1.2	+1.2	-5.1	5.1	-13	13
3600	7.8	-7.9	.5	-.5	-1.2	+1.2	-5.4	5.4	-13	13
3700	8.0	-8.1	.5	-.5	-1.3	+1.3	-5.7	5.7	-14	13
3800	8.2	-8.2	.5	-.5	-1.4	+1.4	-6.0	6.0	-14	14
3900	8.4	-8.4	.6	-.6	-1.5	+1.5	-6.4	6.3	-14	14
4000	8.6	-8.6	.6	-.6	-1.5	+1.5	-6.7	6.7	-14	14

Table F(i)

CHARGE 9 - SHELL 77B

Basic Data

Altitude Sea Level

1	2	3	4	5	6	7	8	9	10
Range	Firing Table elevation	Fuze setting	Correction to Fuze Setting to change Height of Burst by Down 10M	Effect on Range for increase of 1 mil in Firing Table elevation	Fork	Time of flight	Correction to Bearing for drift	Correction to Bearing for 1 knot cross wind	Correction to "Corrector" to change Height of Burst by Down 10M
(X)	(A_E)	(FS)	$\left(\frac{\Delta_C FS}{-10 M Y_s}\right)$	$\left(\frac{\Delta_{FE} X}{+1 \bar{m} A_E}\right)$	(f)	(t)	$(\Delta_C A_d)$	$\left(\frac{\Delta_C A_B}{1 KT W_Z}\right)$	
M	m̄			M	m̄	S	m̄L	m̄	points
4100	36.5			93.0	1	5.8	1.5	.10	
4200	37.6			92.0	1	5.9	1.5	.10	
4300	38.7			91.1	1	6.1	1.6	.10	
4400	39.8			90.3	1	6.3	1.6	.10	
4500	40.9			89.2	1	6.4	1.7	.11	
4600	42.0			88.3	1	6.6	1.7	.11	
4700	43.2			87.6	1	6.8	1.8	.11	
4800	44.3			86.5	1	6.9	1.8	.12	
4900	45.5			85.9	1	7.1	1.9	.12	
5000	46.7			84.8	1	7.3	1.9	.12	
5100	47.8			83.8	1	7.4	2.0	.12	
5200	49.0			83.1	1	7.6	2.0	.13	
5300	50.3			82.2	1	7.8	2.1	.13	
5400	51.5			81.5	1	8.0	2.1	.13	
5500	52.7			80.4	1	8.2	2.2	.14	
5600	54.0			79.6	1	8.3	2.2	.14	
5700	55.2			78.7	1	8.5	2.3	.14	
5800	56.5			77.9	1	8.7	2.4	.14	
5900	57.8			77.9	1	8.9	2.4	.15	
6000	59.1			76.3	1	9.1	2.5	.15	
6100	60.4			75.4	1	9.3	2.5	.15	
6200	61.7			74.7	1	9.5	2.6	.16	
6300	63.1			73.8	1	9.6	2.6	.16	
6400	64.4			73.2	1	9.8	2.7	.16	
6500	65.8			72.3	1	10.0	2.8	.17	
6600	67.2			71.4	1	10.2	2.8	.17	
6700	68.6			70.8	1	10.4	2.9	.17	
6800	70.0			69.9	1	10.6	2.9	.18	
6900	71.5			69.1	1	10.8	3.0	.18	
7000	72.9			68.3	1	11.0	3.1	.18	
7100	74.4			67.6	1	11.2	3.1	.19	
7200	75.9			66.7	1	11.4	3.2	.19	
7300	77.4			65.9	2	11.6	3.2	.19	
7400	78.9			65.3	2	11.9	3.3	.20	
7500	80.5			64.8	2	12.1	3.4	.20	
7600	82.0			63.9	2	12.3	3.4	.20	
7700	83.6			63.1	2	12.5	3.5	.21	
7800	85.2			62.4	2	12.7	3.6	.21	
7900	86.8			61.7	2	12.9	3.6	.21	
8000	88.4			60.9	2	13.1	3.7	.22	

Table F(ii)

CHARGE 9 - SHELL 77B

Corrections to range for non-standard conditions $\Delta_C X$

Altitude Sea Level

1	2	3	4	5	6	7	8	9	10	11
Range	1 M/S Muzzle Velocity (1 M/S V_0)		1 Knot Ballistic Wind (1 KT W_B)		1 % Ballistic Air Temperature (1 % T_B)		1 % Ballistic Density or Drag Coefficient (1 % D_B)		1 □ Projectile Weight (1 □ m_{ej})	
(X)	Decrease (-)	Increase (+)	Head $(W)_-$	Tail $(W)_+$	Decrease (-)	Increase (+)	Decrease (-)	Increase (+)	Decrease (-)	Increase (-)
M	M	M	M	M	M	M	M	M	M	M
4100	8.7	-8.8	.6	-.6	-1.6	+1.6	-7.0	7.0	-14	14
4200	8.9	-9.0	.7	-.7	-1.7	+1.7	-7.4	7.3	-14	14
4300	9.1	-9.2	.7	-.7	-1.8	+1.8	-7.7	7.7	-14	14
4400	9.3	-9.3	.7	-.7	-1.9	+1.9	-8.1	8.1	-14	14
4500	9.5	-9.5	.8	-.8	-2.0	+2.0	-8.4	8.4	-14	14
4600	9.6	-9.7	.8	-.8	-2.1	+2.1	-8.8	8.8	-14	14
4700	9.8	-9.9	.8	-.8	-2.1	+2.2	-9.2	9.2	-15	14
4800	10.0	-10.0	.9	-.9	-2.2	+2.2	-9.6	9.6	-15	14
4900	10.2	-10.2	.9	-.9	-2.3	+2.3	-10.0	10.0	-15	14
5000	10.3	-10.4	1.0	-.9	-2.5	+2.5	-10.4	10.4	-15	14
5100	10.5	-10.6	1.0	-1.0	-2.6	+2.6	-10.8	10.8	-15	14
5200	10.7	-10.7	1.0	-1.0	-2.7	+2.7	-11.2	11.2	-15	14
5300	10.9	-10.9	1.1	-1.1	-2.8	+2.8	-11.7	11.6	-14	14
5400	11.0	-11.1	1.1	-1.1	-2.9	+2.9	-12.1	12.1	-14	14
5500	11.2	-11.2	1.2	-1.2	-3.0	+3.0	-12.6	12.5	-14	14
5600	11.3	-11.4	1.2	-1.2	-3.1	+3.1	-13.0	12.9	-14	14
5700	11.5	-11.6	1.3	-1.2	-3.2	+3.2	-13.5	13.4	-14	14
5800	11.7	-11.7	1.3	-1.3	-3.4	+3.4	-13.9	13.9	-14	14
5900	11.8	-11.9	1.3	-1.3	-3.5	+3.5	-14.4	14.3	-14	14
6000	12.0	-12.0	1.4	-1.4	-3.6	+3.6	-14.9	14.8	-14	14
6100	12.2	-12.2	1.4	-1.4	-3.7	+3.7	-15.4	15.3	-14	14
6200	12.3	-12.4	1.5	-1.5	-3.9	+3.9	-15.9	15.8	-14	13
6300	12.5	-12.5	1.5	-1.5	-4.0	+4.0	-16.4	16.3	-14	13
6400	12.6	-12.7	1.6	-1.6	-4.2	+4.1	-16.9	16.8	-13	13
6500	12.8	-12.8	1.6	-1.6	-4.3	+4.3	-17.4	17.4	-13	13
6600	12.9	-13.0	1.7	-1.7	-4.4	+4.4	-18.0	17.9	-13	13
6700	13.1	-13.1	1.8	-1.7	-4.6	+4.6	-18.5	18.4	-13	13
6800	13.2	-13.3	1.8	-1.8	-4.7	+4.7	-19.1	19.0	-13	12
6900	13.4	-13.4	1.9	-1.9	-4.9	+4.8	-19.6	19.5	-12	12
7000	13.5	-13.6	1.9	-1.9	-5.0	+5.0	-20.2	20.1	-12	12
7100	13.7	-13.7	2.0	-2.0	-5.2	+5.1	-20.7	20.6	-12	12
7200	13.8	-13.9	2.0	-2.0	-5.3	+5.3	-21.3	21.2	-12	12
7300	14.0	-14.0	2.1	-2.1	-5.5	+5.4	-21.9	21.8	-12	11
7400	14.1	-14.1	2.2	-2.1	-5.7	+5.6	-22.5	22.4	-11	11
7500	14.3	-14.3	2.2	-2.2	-5.8	+5.7	-23.1	22.9	-11	11
7600	14.4	-14.4	2.3	-2.3	-6.0	+5.9	-23.7	23.5	-11	10
7700	14.5	-14.6	2.4	-2.3	-6.1	+6.1	-24.3	24.1	-10	10
7800	14.7	-14.7	2.4	-2.4	-6.3	+6.2	-24.9	24.8	-10	10
7900	14.8	-14.8	2.5	-2.5	-6.5	+6.4	-25.5	25.4	-10	10
8000	14.9	-15.0	2.6	-2.5	-6.7	+6.6	-26.2	26.0	-9	9

Table F(i)

CHARGE 9 - SHELL 77B

Basic Data

Altitude Sea Level

1	2	3	4	5	6	7	8	9	10
Range	Firing Table elevation	Fuze setting	Correction to Fuze Setting to change Height of Burst by Down 10M	Effect on Range for increase of 1 mil in Firing Table elevation	Fork	Time of flight	Correction to Bearing for drift	Correction to Bearing for 1 knot cross wind	Correction to "Corrector" to change Height of Burst by Down 10M
(X)	(λ_E)	(FS)	$\left(\Delta_C FS / -10M Y_b\right)$	$\left(\Delta_{EF} X / +1 m\lambda_E\right)$	(f)	(t)	$(\Delta_C A_d)$	$\left(\Delta_C A_R / 1KT W_Z\right)$	
M	m			M	m	S	mL	m	points
8100	90.1			60.2	2	13.4	3.8	.22	
8200	91.8			59.5	2	13.6	3.8	.23	
8300	93.4			59.0	2	13.8	3.9	.23	
8400	95.2			58.1	2	14.0	4.0	.23	
8500	96.9			57.5	2	14.3	4.0	.24	
8600	98.6			56.7	2	14.5	4.1	.24	
8700	100.4			56.2	2	14.7	4.2	.24	
8800	102.2			55.4	2	15.0	4.3	.25	
8900	104.0			55.0	2	15.2	4.3	.25	
9000	105.9			54.3	2	15.5	4.4	.26	
9100	107.7			53.5	2	15.7	4.5	.26	
9200	109.6			52.8	2	15.9	4.6	.26	
9300	111.5			52.3	2	16.2	4.6	.27	
9400	113.4			51.6	2	16.4	4.7	.27	
9500	115.4			50.9	2	16.7	4.8	.28	
9600	117.3			50.3	2	16.9	4.9	.28	
9700	119.3			49.5	2	17.2	4.9	.28	
9800	121.4			49.1	2	17.4	5.0	.29	
9900	123.4			48.6	2	17.7	5.1	.29	
10000	125.5			47.9	3	18.0	5.2	.30	
10100	127.6			47.3	3	18.2	5.3	.30	
10200	129.7			46.8	3	18.5	5.4	.30	
10300	131.9			46.2	3	18.8	5.4	.31	
10400	134.1			45.6	3	19.0	5.5	.31	
10500	136.3			45.0	3	19.3	5.6	.32	
10600	138.5			44.3	3	19.6	5.7	.32	
10700	140.8			43.9	3	19.9	5.8	.33	
10800	143.1			43.3	3	20.2	5.9	.33	
10900	145.4			42.8	3	20.4	6.0	.34	
11000	147.7			42.2	3	20.7	6.1	.34	
11100	150.1			41.8	3	21.0	6.2	.34	
11200	152.5			41.1	3	21.3	6.3	.35	
11300	155.0			40.8	3	21.6	6.3	.35	
11400	157.4			40.2	3	21.9	6.4	.36	
11500	160.0			39.6	3	22.2	6.5	.36	
11600	162.5			39.1	4	22.5	6.6	.37	
11700	165.1			38.7	4	22.8	6.7	.37	
11800	167.7			38.1	4	23.1	6.8	.38	
11900	170.3			37.7	4	23.4	6.9	.38	
12000	173.0			37.2	4	23.7	7.0	.39	

Table F(ii)

CHARGE 9 - SHELL 77B

Corrections to range for non-standard conditions $\Delta_C X$

Altitude Sea Level

1	2	3	4	5	6	7	8	9	10	11
Range	1 M/S Muzzle Velocity (1 M/S V_0)		1 Knot Ballistic Wind (1 KT W_B)		1 % Ballistic Air Temperature (1 % T_B)		1 % Ballistic Density or Drag Coefficient (1 % D_B)		1 □ Projectile Weight (1 □ E_T)	
(X)	Decrease (−)	Increase (+)	Head (W) −	Tail (W) +	Decrease (−)	Increase (+)	Decrease (−)	Increase (+)	Decrease (−)	Increase (+)
M	M	M	M	M	M	M	M	M	M	M
8100	15.1	-15.1	2.6	-2.6	-6.8	+6.7	-26.8	26.6	-9	9
8200	15.2	-15.2	2.7	-2.7	-7.0	+6.9	-27.5	27.3	-9	9
8300	15.4	-15.4	2.8	-2.7	-7.2	+7.1	-28.1	27.9	-8	8
8400	15.5	-15.5	2.8	-2.8	-7.4	+7.2	-28.8	28.6	-8	8
8500	15.6	-15.6	2.9	-2.9	-7.5	+7.4	-29.5	29.2	-8	7
8600	15.7	-15.8	3.0	-3.0	-7.7	+7.6	-30.1	29.9	-7	7
8700	15.9	-15.9	3.1	-3.0	-7.9	+7.8	-30.8	30.6	-7	7
8800	16.0	-16.0	3.1	-3.1	-8.1	+7.9	-31.5	31.3	-7	6
8900	16.1	-16.1	3.2	-3.2	-8.3	+8.1	-32.2	32.0	-6	6
9000	16.3	-16.3	3.3	-3.3	-8.5	+8.3	-32.9	32.7	-6	5
9100	16.4	-16.4	3.4	-3.4	-8.7	+8.5	-33.6	33.4	-5	5
9200	16.5	-16.5	3.5	-3.4	-8.9	+8.7	-34.3	34.1	-5	5
9300	16.6	-16.6	3.5	-3.5	-9.1	+8.9	-35.1	34.8	-4	4
9400	16.8	-16.8	3.6	-3.6	-9.3	+9.1	-35.8	35.5	-4	4
9500	16.9	-16.9	3.7	-3.7	-9.5	+9.3	-36.5	36.2	-3	3
9600	17.0	-17.0	3.8	-3.8	-9.7	+9.5	-37.3	37.0	-3	3
9700	17.1	-17.1	3.9	-3.9	-9.9	+9.6	-38.0	37.7	-2	2
9800	17.3	-17.3	4.0	-4.0	-10.1	+9.8	-38.8	38.5	-2	2
9900	17.4	-17.4	4.1	-4.1	-10.3	+10.0	-39.5	39.2	-2	1
10000	17.5	-17.5	4.2	-4.1	-10.5	+10.3	-40.3	40.0	-1	1
10100	17.6	-17.6	4.3	-4.2	-10.7	+10.5	-41.1	40.8	0	0
10200	17.7	-17.7	4.4	-4.3	-10.9	+10.7	-41.9	41.5	0	0
10300	17.8	-17.8	4.4	-4.4	-11.1	+10.9	-42.7	42.3	0	0
10400	18.0	-18.0	4.5	-4.5	-11.4	+11.1	-43.5	43.1	0	-1
10500	18.1	-18.1	4.6	-4.6	-11.6	+11.3	-44.3	43.9	1	-1
10600	18.2	-18.2	4.7	-4.7	-11.8	+11.5	-45.1	44.7	1	-2
10700	18.3	-18.3	4.8	-4.8	-12.0	+11.8	-45.9	45.5	2	-2
10800	18.4	-18.4	5.0	-4.9	-12.3	+12.0	-46.7	46.3	2	-3
10900	18.5	-18.5	5.1	-5.0	-12.5	+12.2	-47.6	47.2	3	-3
11000	18.7	-18.6	5.2	-5.1	-12.7	+12.4	-48.4	48.0	4	-4
11100	18.8	-18.7	5.3	-5.2	-13.0	+12.7	-49.3	48.9	4	-5
11200	18.9	-18.9	5.4	-5.4	-13.2	+12.9	-50.1	49.7	5	-5
11300	19.0	-19.0	5.5	-5.5	-13.5	+13.1	-51.0	50.5	5	-6
11400	19.1	-19.1	5.6	-5.6	-13.7	+13.3	-51.8	51.4	6	-6
11500	19.2	-19.2	5.7	-5.7	-13.9	+13.5	-52.7	52.2	7	-7
11600	19.3	-19.3	5.8	-5.8	-14.2	+13.7	-53.6	53.1	7	-8
11700	19.4	-19.4	5.9	-5.9	-14.4	+13.9	-54.5	54.0	8	-8
11800	19.5	-19.5	6.1	-6.0	-14.7	+14.1	-55.3	54.8	9	-9
11900	19.7	-19.6	6.2	-6.2	-14.9	+14.3	-56.2	55.7	9	-10
12000	19.8	-19.7	6.3	-6.3	-15.1	+14.5	-57.1	56.5	10	-10

Table F(i)

CHARGE 9 - SHELL 77B

Basic Data

Altitude Sea Level

1	2	3	4	5	6	7	8	9	10
Range	Firing Table elevation	Fuze setting	Correction to Fuze Setting to change Height of Burst by Down 10M	Effect on Range for increase of 1 mil in Firing Table elevation	Fork	Time of flight	Correction to Bearing for drift	Correction to Bearing for 1 knot cross wind	Correction to "Corrector" to change Height of Burst by Down 10M
(X)	(A_E)	(FS)	$\left(\frac{\Delta_C FS}{-10 MY_S}\right)$	$\left(\frac{\Delta_{EF} X}{+1 \text{mil} A_E}\right)$	(f)	(t)	($\Delta_C A_d$)	$\left(\frac{\Delta_C A_B}{1 KT W_Z}\right)$	
M	m			M	mil	S	mil	m	points
12100	175 7			36 7	4	24.1	7.1	39	
12200	178 4			36 3	4	24 4	7 2	40	
12300	181 2			35.8	4	24.7	7 3	40	
12400	184 0			35.3	4	25.0	7 4	41	
12500	186 9			34 9	4	25.3	7 5	41	
12600	189 7			34 6	4	25.7	7 6	.41	
12700	192.6			34 2	4	26.0	7.7	.42	
12800	195.6			33 8	5	26.3	7.8	.42	
12900	198 6			33.5	5	26.6	7 9	43	
13000	201.6			33.1	5	27.0	8 0	43	
13100	204 6			32.7	5	27.3	8.0	.44	
13200	207 7			32.3	5	27 6	8.1	44	
13300	210 8			32 0	5	28.0	8.2	45	
13400	214 0			31.7	5	28.3	8.3	45	
13500	217 1			31.3	5	28.6	8.4	.46	
13600	220 3			31.0	5	29 0	8 5	46	
13700	223.6			30.7	5	29.3	8.6	46	
13800	226 9			30.3	5	29.7	8.7	47	
13900	230 2			30.1	6	30.0	8 8	47	
14000	233.5			29 7	6	30.3	8 9	48	
14100	236 9			29 5	6	30 7	[illegible]	48	
14200	240 3			29 2	6	[illegible]	9 2	49	
14300	243 8			28 9	[illegible]	31 4	9 3	49	
14400	247 2			28 6	6	31.7	9 4	49	
14500	250 7			[illegible]	6	32.1	9 5	50	
14600	[illegible]			28.0	6	32.4	9 6	50	
14700	[illegible]			27.9	6	32.8	9 7	.51	
[illegible]	261 5			27 6	6	[illegible]	9 8	51	
14900	265.1			27.2	6	33 5	9.9	.52	
15000	268 8			27.0	7	33.9	10.0	.52	
15100	272.5			26 9	7	34.2	10.1	52	
15200	276 2			26 6	7	34.6	10.2	.53	
15300	280.0			26.3	7	35.0	10.3	.53	
15400	283.8			26.0	7	35.3	10.5	.54	
15500	287.7			25 9	7	35.7	10.6	.54	
15600	291.6			25.6	7	36 1	10.7	.54	
15700	295.5			25.4	7	36.5	10.8	.55	
15800	299.4			25.1	7	36.8	10.9	.55	
15900	303.4			25 0	8	37.2	11.0	.56	
16000	307 4			24.7	8	37.6	11.2	.56	

Table F(ii)

CHARGE 9 - SHELL 77B

Corrections to range for non-standard conditions $\Delta_C X$

Altitude Sea Level

1	2	3	4	5	6	7	8	9	10	11
Range	1 M/S Muzzle Velocity ($1 M/S V_O$)		1 Knot Ballistic Wind ($1 KT W_B$)		1 % Ballistic Air Temperature ($1\% T_B$)		1 % Ballistic Density or Drag Coefficient ($1\% D_B$)		1 □ Projectile Weight ($1 □ m_{oj}$)	
(X)	Decrease (-)	Increase (+)	Head (W)_	Tail (W)+	Decrease (-)	Increase (+)	Decrease (-)	Increase (+)	Decrease (-)	Increase (+)
M	M	M	M	M	M	M	M	M	M	M
12100	19.9	-19.8	6.4	-6 4	-15 3	+14.7	-57.9	57.3	10	-11
12200	20.0	-20 0	6 5	-6 5	-15 6	+14 8	-58 8	58.2	11	-12
12300	20.1	-20 1	6.7	-6.7	-15.8	+14.9	-59 6	59 0	12	-12
12400	20 2	-20 2	6.8	-6 8	-16 0	+15.1	-60.5	59 8	12	-13
12500	20.3	-20 3	6 9	-6 9	-16 1	+15.2	-61.3	60.7	13	-14
12600	20 4	-20 4	7 1	-7 0	-16 3	+15 3	-62.1	61.5	13	-14
12700	20.5	-20.5	7.2	-7 2	-16 5	+15 4	-63 0	62.3	14	-15
12800	20 6	-20 6	7.3	-7.3	-16.6	+15.5	-63 8	63 1	15	-15
12900	20.7	-20.7	7.5	-7.5	-16.7	+15.5	-64.6	63 8	15	-16
13000	20.8	-20.8	7 6	-7.6	-16 8	+15 6	-65 4	64 6	16	-17
13100	20.9	-20.9	7 8	-7 7	-17 0	+15 6	-66.2	65.4	16	-17
13200	21.0	-21.0	7.9	-7 9	-17 0	+15.6	-67.0	66.1	17	-18
13300	21.1	-21.1	8 0	-8.0	-17 1	+15 7	-67.7	66.9	17	-18
13400	21.2	-21.2	8 2	-8 2	-17 2	+15.7	-68.5	67 6	18	-19
13500	21.3	-21 3	8.3	-8.3	-17 2	+15.7	-69.3	68.4	18	-19
13600	21.4	-21.4	8.5	-8.5	-17.3	+15 6	-70.0	69.1	19	-20
13700	21.5	-21.5	8.6	-8 6	-17.3	+15.6	-70.8	69 8	19	-20
13800	21 6	-21.6	8 8	-8.8	-17.3	+15.6	-71.5	70 6	20	-21
13900	21.7	-21.7	8.9	-8.9	-17.3	+15.5	-72.2	71.3	20	-21
14000	21.8	-21.8	9 1	-9 1	-17 3	+15.5	-73.0	72.0	21	-22
14100	21.9	-21.9	9 2	-9 2	-17.3	+15 4	-73.7	72.7	21	-22
14200	22.0	-22.0	9 4	-9 4	-17.3	+15 4	-74 4	73 4	22	-23
14300	22.1	-22.1	9 6	-9.5	-17.3	+15.3	-75.1	74.1	22	-23
14400	22.2	-22.2	9 7	-9.7	-17 2	+15 2	-75 8	74.7	23	-24
14500	22.3	-22.3	9 9	-9 9	-17.1	+15 1	-76.5	75.4	23	-24
14600	22.4	-22.4	10 0	-10 0	-17 1	+15 0	-77.2	76.1	24	-25
14700	22.5	-22.5	10.2	-10.2	-17 0	+14 8	-77 9	76 8	24	-25
14800	22.6	-22.6	10 4	-10.4	-16.9	+14 7	-78.5	77 4	25	-26
14900	22.7	-22.7	10.5	-10 5	-16 8	+14 6	-79.2	78 1	25	-26
15000	22.8	-22.8	10 7	-10.7	-16 7	+14.5	-79 9	78 7	26	-27
15100	22.9	-22.9	10 9	-10.9	-16.6	+14 3	-80 6	79.4	26	-27
15200	23 0	-23.0	11.0	-11.0	-16.5	+14 2	-81.2	80 0	27	-28
15300	23.1	-23 1	11 2	-11.2	-16 4	+14 0	-81.9	80.7	27	-28
15400	23.2	-23.2	11.4	-11 4	-16 3	+13.8	-82.5	81.3	27	-29
15500	23.3	-23 3	11.5	-11.5	-16 1	+13 6	-83 1	81.9	28	-29
15600	23.4	-23 4	11.7	-11.7	-16 0	+13.5	-83 8	82.6	28	-29
15700	23.5	-23.5	11 9	-11.9	-15 8	+13.3	-84 4	83.2	29	-30
15800	23 6	-23 6	12.1	-12.1	-15.6	+13.1	-85.0	83.8	29	-30
15900	23.7	-23.7	12.2	-12.2	-15.5	+12.9	-85 7	84.5	29	-31
16000	23.3	-23.8	12.4	-12.4	-15 3	+12.7	-86.3	85.1	30	-31

Table F(i)

CHARGE 9 - SHELL 77B

Basic Data

Altitude Sea Level

1	2	3	4	5	6	7	8	9	10
Range	Firing Table Elevation	Fuze Setting	Correction to Fuze [illegible]	Effect on Range for increase of 1 mil in firing angle	Fork	Time of flight	correction to Bearing for drift	Correction to Bearing for 1 Knot crosswind	Correction to [illegible]
(λ)	(A_E)	(FS)	(ΔFS / $-10 W_b$)	($\Delta\lambda$ / $+1 \Delta A_F$)	(f)	(t)	($\Delta_C A_d$)	($\Delta_C A_h$ / 1 KT W_z)	
M	mil			M	mil	S	mil L	mil	points
16100	311.5			24.6	8	38.0	11.3	.56	
16200	315.6			24.3	8	38.4	11.4	.57	
16300	319.7			24.2	8	38.7	11.5	.57	
16400	323.9			23.9	8	39.1	11.7	.58	
16500	328.1			23.7	8	39.5	11.8	.58	
16600	332.3			23.6	8	39.9	11.9	.58	
16700	336.6			23.3	8	40.3	12.0	.59	
16800	340.9			23.0	9	40.7	12.2	.59	
16900	345.2			22.9	9	41.1	12.3	.59	
17000	349.6			22.7	9	41.5	12.4	.60	
17100	354.0			22.5	9	41.9	12.6	.60	
17200	358.5			22.3	9	42.3	12.7	.61	
17300	363.0			22.1	9	42.7	12.8	.61	
17400	367.5			21.8	9	43.1	13.0	.61	
17500	372.1			21.7	9	43.5	13.1	.62	
17600	376.8			21.5	10	43.9	13.2	.62	
17700	381.4			21.4	10	44.4	13.4	.62	
17800	386.1			21.1	10	44.8	13.5	.63	
17900	390.9			20.9	10	45.2	13.6	.63	
18000	395.7			20.7	10	45.6	13.8	.64	
18100	400.5			20.5	10	46.1	13.9	.64	
18200	405.4			20.4	10	46.5	14.1	.64	
18300	410.4			20.2	11	46.9	14.2	.65	
18400	415.4			19.9	11	47.4	14.4	.65	
18500	420.4			19.7	11	47.8	14.5	.65	
18600	425.5			19.5	11	48.3	14.7	.66	
18700	430.7			19.3	11	48.7	14.8	.66	
18800	435.9			19.2	11	49.2	15.0	.67	
18900	441.1			18.9	12	49.6	15.2	.67	
19000	446.4			18.7	12	50.1	15.3	.67	
19100	451.8			18.5	12	50.6	15.5	.68	
19200	457.2			18.3	12	51.0	15.6	.68	
19300	462.7			18.1	12	51.5	15.8	.68	
19400	468.3			17.9	13	52.0	16.0	.69	
19500	473.9			17.7	13	52.5	16.1	.69	
19600	479.6			17.4	13	53.0	16.3	.70	
19700	485.3			17.3	13	53.5	16.5	.70	
19800	491.1			17.1	13	54.0	16.7	.70	
19900	497.0			16.8	14	54.5	16.9	.71	
20000	503.0			16.6	14	55.0	17.0	.71	

Table F(ii)

CHARGE 9 - SHELL 77B

Corrections to range for non-standard conditions $\Delta_C \lambda$

Altitude Sea Level

1	2	3	4	5	6	7	8	9	10	11
	1 M/S Muzzle Velocity (1 M/s V_0)		1 Knot Ballistic Wind (1 Knot W_B)		1% Ballistic Air Temperature (1% T_B)		1% Ballistic Density or Drag Coefficient (1% D_B)		1 □ Projectile Weight (1 □ mp)	
(λ)	Decrease (-)	Increase (+)	Head (W)−	Tail (W)+	Decrease (-)	Increase (+)	Decrease (-)	Increase (+)	Decrease (-)	Increase (+)
M	M	M	M	M	M	M	M	M	M	M
16100	23.9	-23.9	12.6	-12.6	-15.1	+12.5	-86.9	85.7	30	-31
16200	24.0	-24.0	12.8	-12.8	-14.9	+12.2	-87.5	86.3	31	-32
16300	24.1	-24.1	13.0	-13.0	-14.7	+12.0	-88.2	86.9	31	-32
16400	24.2	-24.2	13.2	-13.2	-14.5	+11.8	-88.8	87.5	31	-33
16500	24.3	-24.3	13.3	-13.3	-14.3	+11.5	-89.4	88.1	32	-33
16600	24.4	-24.4	13.5	-13.5	-14.1	+11.3	-90.0	88.7	32	-33
16700	24.5	-24.5	13.7	-13.7	-13.8	+11.0	-90.6	89.2	32	-34
16800	24.6	-24.6	13.9	-13.9	-13.6	+10.7	-91.2	89.8	33	-34
16900	24.7	-24.7	14.1	-14.1	-13.4	+10.5	-91.8	90.4	33	-35
17000	24.8	-24.8	14.3	-14.3	-13.1	+10.2	-92.4	91.0	34	-35
17100	24.9	-24.9	14.5	-14.5	-12.9	+9.9	-93.0	91.5	34	-35
17200	25.0	-25.0	14.7	-14.7	-12.6	+9.6	-93.6	92.1	34	-36
17300	25.1	-25.1	14.9	-14.9	-12.3	+9.3	-94.2	92.7	35	-36
17400	25.2	-25.2	15.1	-15.1	-12.1	+9.0	-94.8	93.3	35	-37
17500	25.3	-25.3	15.3	-15.3	-11.8	+8.7	-95.3	93.9	35	-37
17600	25.4	-25.4	15.5	-15.5	-11.5	+8.4	-95.9	94.4	36	-37
17700	25.5	-25.5	15.7	-15.7	-11.2	+8.1	-96.5	95.0	36	-38
17800	25.6	-25.6	15.9	-15.9	-10.9	+7.8	-97.1	95.5	36	-38
17900	25.7	-25.7	16.1	-16.1	-10.6	+7.5	-97.7	96.1	[illegible]	[illegible]
18000	25.8	-25.8	16.3	-16.3	-10.3	+7.2	-98.3	96.6	37	-39
18100	25.9	-26.0	16.5	-16.5	-10.0	+6.8	-98.8	97.2	37	-39
18200	26.0	-26.1	16.7	-16.7	-9.7	+6.5	-99.4	97.7	38	-39
18300	26.1	-26.2	16.9	-16.9	-9.4	+6.1	-99.9	[illegible]	[illegible]	-40
18400	26.2	-26.3	17.1	-17.1	-9.0	+5.8	-100.4	98.9	38	-40
18500	26.3	-26.4	17.3	-17.4	-8.7	+5.4	-101.0	99.4	39	-40
18600	26.4	-26.5	17.5	-17.6	-8.4	+5.1	-101.5	100.0	39	-41
18700	26.5	-26.6	17.7	-17.8	-8.0	+4.7	-102.0	100.5	39	-41
18800	26.6	-26.7	18.0	-18.0	-7.7	+4.3	-102.6	101.1	40	-41
18900	26.7	-26.8	18.2	-18.2	-7.3	+3.9	-103.2	101.6	40	-42
19000	26.9	-26.9	18.4	-18.4	-6.9	+3.6	-103.7	102.1	40	-42
19100	27.0	-27.0	18.6	-18.7	-6.6	+3.2	-104.3	102.7	40	-42
19200	27.1	-27.1	18.8	-18.9	-6.2	+2.8	-104.8	103.2	41	-43
19300	27.2	-27.2	19.1	-19.1	-5.8	+2.4	-105.4	103.7	41	-43
19400	27.3	-27.3	19.3	-19.4	-5.4	+2.0	-105.9	104.2	41	-43
19500	27.4	-27.4	19.5	-19.6	-5.0	+1.6	-106.5	104.7	42	-44
19600	27.5	-27.5	19.8	-19.8	-4.6	+1.1	-107.0	105.2	42	-44
19700	27.6	-27.6	20.0	-20.1	-4.1	+.7	-107.5	105.8	42	-44
19800	27.7	-27.7	20.2	-20.3	-3.7	+.3	-108.0	106.3	42	-44
19900	27.8	-27.8	20.5	-20.5	-3.3	-.2	-108.6	106.8	43	-45
20000	27.9	-27.9	20.7	-20.8	-2.8	-.6	-109.1	107.3	43	-45

Table F(i)

CHARGE 9 - SHELL 77B

Basic Data

Altitude Sea Level

1	2	3	4	5	6	7	8	9	10
Range	Firing Table elevation	Fuze setting	Correction to Fuze Setting to change Height of Burst by Down 10M	Effect on Range for increase of 1 mil in Firing Table elevation	Fork	Time of flight	Correction to Bearing for drift	Correction to Bearing for 1 knot cross wind	Correction to "Corrector" to change Height of Burst by Down 10M
(X)	(A_E)	(FS)	$\left(\frac{\Delta_C FS}{-10MY_b}\right)$	$\left(\frac{\Delta_{EF} X}{+1 \text{ mil } A_E}\right)$	(f)	(t)	$(\Delta_C A_d)$	$\left(\frac{\Delta_C A_B}{1KTW_Z}\right)$	
M	mil			M	mil	S	miL	mil	points
20100	509.1			16.4	14	55.5	17.2	.72	
20200	515.2			16.2	14	56.0	17.4	.72	
20300	521.5			16.0	15	56.6	17.6	.72	
20400	527.8			15.7	15	57.1	17.8	.73	
20500	534.2			15.4	15	57.6	18.0	.73	
20600	540.8			15.2	16	58.2	18.2	.74	
20700	547.4			14.9	16	58.8	18.4	.74	
20800	554.2			14.6	16	59.3	18.7	.74	
20900	561.0			14.4	17	59.9	18.9	.75	
21000	568.0			14.1	17	60.5	19.1	.75	
21100	575.2			13.8	18	61.1	19.4	.76	
21200	582.5			13.6	18	61.7	19.6	.76	
21300	589.9			13.3	18	62.4	19.9	.77	
21400	597.6			12.9	19	63.0	20.1	.77	
21500	605.4			12.8	19	63.7	20.4	.78	
21600	613.4			12.3	20	64.4	20.7	.78	
21700	621.6			12.0	21	65.1	20.9	.79	
21800	630.1			11.8	21	65.8	21.3	.79	
21900	638.7			11.3	22	66.5	21.6	.80	
22000	647.8			10.9	23	67.3	21.9	.80	
22100	657.1			10.8	23	68.0	22.2	.81	
22200	666.6			10.1	25	68.9	22.6	.82	
22300	676.8			9.5	27	69.7	23.0	.82	
22400	687.5			9.4	27	70.6	23.4	.83	
22500	698.7			8.6	30	71.5	23.9	.83	
22600	710.9			7.8	33	72.6	24.4	.84	
22700	724.1			7.3	36	73.7	25.0	.85	
22800	738.6			6.6	40	74.9	25.7	.86	
22900	754.8			6.1	44	76.2	26.5	.87	
23000	774.9			4.2	66	77.9	27.6	.89	
23100	802.3			[illegible]	88	80.1	29.2	.91	
23100	884.1			-3.4	-90	86.5	34.8	.97	
23000	909.9			-4.5	-67	88.5	36.9	.99	
22900	927.9			-6.1	-50	89.8	38.5	1.00	
22300	942.5			-7.3	-42	90.8	39.8	1.01	
22700	954.9			-8.5	-36	91.7	41.0	1.03	
22600	966.2			-9.3	-33	92.5	42.1	1.04	
22500	976.1			-10.4	-30	93.2	43.1	1.05	
22400	985.5			-11.0	-28	93.9	44.0	1.06	
22300	994.1			-12.1	-26	94.4	45.0	1.07	

Table F(ii)

CHARGE 9 - SHELL 77B

Corrections to range for non-standard conditions $\Delta_C X$

Altitude Sea Level

1	2	3	4	5	6	7	8	9	10	11
Range	1 M/S Muzzle Velocity ($1 M/S V_0$)		1 Knot Ballistic Wind ($1 KT W_B$)		1 % Ballistic Air Temperature ($1\% T_B$)		1% Ballistic Density or Drag Coefficient ($1\% D_B$)		1 ☐ Projectile Weight ($1☐ m_{pj}$)	
(X)	Decrease (-)	Increase (+)	Head (W)	Tail (W) +	Decrease (-)	Increase (+)	Decrease (-)	Increase (+)	Decrease (-)	Increase (+)
M	M	M	M	M	M	M	M	M	M	M
20100	28.0	-28.0	21.0	-21.0	-2.3	-1.1	-109.6	107.8	43	-45
20200	28.1	-28.2	21.2	-21.3	-1.8	-1.6	-110.1	108.4	44	-46
20300	28.2	-28.3	21.5	-21.5	-1.3	-2.1	-110.7	108.9	44	-46
20400	28.3	-28.4	21.7	-21.8	-.8	-2.6	-111.2	109.4	44	-46
20500	28.4	-28.5	22.0	-22.1	-.3	-3.1	-111.7	109.9	44	-47
20600	28.5	-28.6	22.2	-22.3	.2	-3.6	-112.2	110.3	45	-47
20700	28.6	-28.7	22.5	-22.6	.7	-4.2	-112.7	110.8	45	-47
20800	28.7	-28.8	22.8	-22.9	1.3	-4.8	-113.1	111.2	45	-47
20900	28.8	-28.9	23.0	-23.1	1.9	-5.3	-113.6	111.7	45	-48
21000	28.9	-29.0	23.3	-23.4	2.4	-5.9	-114.1	112.1	45	-48
21100	29.0	-29.1	23.6	-23.7	3.0	-6.4	-114.6	112.5	46	-48
21200	29.1	-29.2	23.9	-24.0	3.6	-7.0	-115.0	112.9	46	-48
21300	29.1	-29.3	24.2	-24.3	4.3	-7.5	-115.4	113.2	46	-49
21400	29.2	-29.4	24.5	-24.6	4.9	-8.1	-115.9	113.6	46	-49
21500	29.3	-29.5	24.8	-24.9	5.6	-8.6	-116.3	114.0	46	-49
21600	29.4	-29.6	25.1	-25.2	6.2	-9.1	-116.7	114.5	46	-49
21700	29.5	-29.7	25.4	-25.5	6.9	-9.5	-117.2	115.0	47	-49
[illegible]	29.6	-29.8	25.7	-25.8	7.7	-9.9	-117.5	115.5	47	-50
21900	29.7	-29.8	26.0	-26.1	8.6	-10.3	-117.6	116.1	48	-49
22000	29.9	-29.9	26.4	-26.4	9.5	-10.7	-117.9	116.7	48	-49
22100	30.0	-30.0	26.7	-26.8	10.3	-11.0	-118.3	117.5	49	-49
22200	30.2	-30.1	27.1	-27.1	10.9	-11.2	-118.9	118.3	49	-50
[illegible]	30.3	-30.3	27.4	-27.4	11.5	-11.4	-119.6	119.1	50	-50
22400	30.4	-30.3	27.8	-27.8	12.4	-11.7	-119.9	119.5	50	-50
22500	30.4	-30.3	28.1	-28.1	13.6	-12.3	-119.7	119.3	50	-50
22600	30.5	-30.4	28.5	-28.5	14.4	-12.6	-120.2	119.7	50	-50
22700	30.7	-30.7	28.9	-28.8	14.7	-12.5	-121.3	120.8	50	-51
[illegible]	31.1	-31.0	29.3	-29.2	14.6	-12.2	-123.2	122.6	51	-52
[illegible]	31.6	-31.5	29.7	-29.6	14.0	-11.5	-125.8	125.0	53	-54
[illegible]	32.1	-32.1	30.2	-30.0	12.9	-10.5	-129.1	128.1	54	-56
23100	32.8	-32.8	30.6	-30.5	11.4	-9.3	-133.1	131.8	57	-58
[illegible]	34.7	-34.9	31.6	-31.5	3.8	-4.0	-142.4	140.6	61	-63
23000	34.9	-35.2	31.7	-31.5	2.2	-2.8	-143.5	141.7	61	-64
22900	35.?	-35.5	31.8	-31.6	.8	-1.7	-144.5	142.6	61	-64
[illegible]	35.4	-35.7	31.8	-31.6	-.5	-.7	-145.3	143.1	62	-65
22700	35.6	-36.0	31.8	-31.7	-1.7	+.2	-146.1	144.1	62	-65
[illegible]	35.8	-36.2	31.9	-31.7	-2.8	+1.1	-146.7	144.7	62	-65
22500	35.9	-36.3	31.9	-31.7	-3.8	+1.9	-147.3	145.2	62	-65
22400	36.0	-36.5	31.9	-31.7	-4.8	+2.7	-147.7	145.6	62	-65
22300	36.1	-36.6	31.9	-31.7	-5.5	+3.3	-148.1	145.3	52	-65

Table F(i)

CHARGE 9 - SHELL 77B

Basic Data

Altitude Sea Level

1	2	3	4	5	6	7	8	9	10
Range	Firing Table elevation	Fuze setting	Correction to Fuze Setting to change Height of Burst by Down 10M	Effect on Range for increase of 1 mil in Firing Table elevation	Fork	Time of flight	Correction to Bearing for drift	Correction to Bearing for 1 knot cross wind	Correction to "Corrector" to change Height of Burst by Down 10M
(X)	(A_E)	(FS)	($\Delta_C FS$ / -10M$_t$)	($\Delta_{FT} X$ / +1 mil A_E)	(f)	(t)	($\Delta_C A_d$)	($\Delta_C A_B$ / 1KT W_Z)	
M	mil			M	mil	S	mil L	mil	points
22200	1002.2			-12.6	-25	95.0	45.8	1.08	
22100	1010.0			-13.2	-24	95.5	46.7	1.09	
22000	1017.4			-14.0	-22	96.0	47.5	1.10	
21900	1024.4			-14.6	-21	96.4	48.3	1.11	
21800	1031.1			-15.1	-21	96.9	49.1	1.12	
21700	1037.6			-15.8	-20	97.3	49.9	1.13	
21600	1043.8			-16.4	-19	97.7	50.6	1.13	
21500	1049.9			-16.9	-18	98.1	51.4	1.14	
21400	1055.7			-17.5	-18	98.4	52.1	1.15	
21300	1061.3			-18.0	-17	98.8	52.8	1.16	
21200	1066.8			-18.5	-17	99.1	53.6	1.17	
21100	1072.1			-19.0	-16	99.4	54.3	1.17	
21000	1077.3			-19.6	-16	99.7	55.0	1.18	
20900	1082.4			-19.9	-15	100.0	55.7	1.19	
20800	1087.3			-20.2	-15	100.3	56.4	1.20	
20700	1092.1			-21.1	-14	100.6	57.1	1.20	
20600	1096.8			-21.4	-14	100.9	57.8	1.21	
20500	1101.4			-22.0	-14	101.1	58.4	1.22	
20400	1105.9			-22.5	-13	101.4	59.1	1.23	
20300	1110.3			-22.6	-13	101.6	59.8	1.24	
20200	1114.6			-23.1	-13	101.9	60.5	1.24	
20100	1118.9			-23.7	-13	102.1	61.2	1.25	
20000	1123.0			-24.2	-12	102.3	61.9	1.26	
19900	1127.1			-24.5	-12	102.6	62.5	1.27	
19800	1131.2			-25.0	-12	102.8	63.2	1.28	
19700	1135.1			-25.4	-12	103.0	63.9	1.28	
19600	1139.0			-25.8	-11	103.2	64.6	1.29	
19500	1142.9			-26.3	-11	103.4	65.2	1.30	
19400	1146.6			-26.3	-11	103.6	65.9	1.31	
19300	1150.4			-27.1	-11	103.8	66.6	1.32	
19200	1154.0			-27.7	-11	104.0	67.3	1.33	
19100	1157.7			-28.1	-10	104.2	68.0	1.33	
19000	1161.2			-28.1	-10	104.4	68.6	1.34	
18900	1164.8			-28.4	-10	104.5	69.3	1.35	
18800	1168.2			-29.0	-10	104.7	70.0	1.36	
18700	1171.7			-29.3	-10	104.9	70.7	1.37	
18600	1175.1			-29.5	-10	105.1	71.4	1.38	
18500	1178.4			-30.1	-9	105.2	72.1	1.39	
18400	1181.7			-30.3	-9	105.4	72.8	1.40	
18300	1185.0			-30.8	-9	105.5	73.4	1.41	

Table F(ii)

CHARGE 9 - SHELL 77B

Corrections to range for non-standard conditions $\Delta_C X$

Altitude Sea Level

1	2	3	4	5	6	7	8	9	10	11
Range	1 M/S Muzzle Velocity		1 Knot Ballistic Wind		1% Ballistic Air Temperature		1% Ballistic Density or Drag Coefficient		10 Projectile Weight	
	(1 M/S V_O)		(1 KT W_R)		(1% T_B)		(1% D_B)		(10 m_{p})	
(X)	Decrease (-)	Increase (+)	Head (W) -	Tail (W) +	Decrease (-)	Increase (+)	Decrease (-)	Increase (+)	Decrease (-)	Increase (+)
M	M	M	M	M	M	M	M	M	M	M
22200	36.2	-36.7	31.9	-31.6	-6.1	+3.8	-148.3	145.9	62	-66
22100	36.3	-36.8	31.8	-31.6	-6.6	+4.3	-148.4	146.0	62	-66
22000	36.3	-36.9	31.8	-31.5	-7.2	+4.7	-148.4	146.0	61	-66
21900	36.4	-37.0	31.8	-31.5	-7.6	+5.2	-148.4	145.9	61	-65
21800	36.4	-37.0	31.7	-31.4	-8.1	+5.6	-148.4	145.9	61	-65
21700	36.4	-37.0	31.7	-31.4	-8.5	+6.0	-148.2	145.7	61	-65
21600	36.4	-37.0	31.6	-31.3	-8.8	+6.4	-147.9	145.6	61	-65
21500	36.4	-37.0	31.5	-31.3	-9.2	+6.7	-147.6	145.4	61	-64
21400	36.4	-37.0	31.5	-31.2	-9.5	+7.1	-147.3	145.2	60	-64
21300	36.3	-36.9	31.4	-31.2	-9.8	+7.4	-147.0	144.9	60	-64
21200	36.3	-36.9	31.4	-31.1	-10.1	+7.7	-146.6	144.6	60	-64
21100	36.3	-36.9	31.3	-31.1	-17.9	+7.9	146.2	144.2	60	-63
21000	36.2	-36.8	31.3	-31.0	-10.6	+8.2	-145.9	143.8	59	-63
20900	36.1	-36.7	31.2	-30.9	-10.9	+8.4	-145.5	143.4	59	-63
20800	36.1	-36.7	31.1	-30.9	-11.1	+8.6	-145.0	142.9	59	-63
20700	36.0	-36.6	31.1	-30.8	-11.4	+8.8	-144.6	142.5	58	-62
20600	35.9	-36.6	31.0	-30.7	-11.6	+9.0	-144.2	142.0	58	-62
20500	35.9	-36.5	31.0	-30.7	-11.8	+9.1	-143.6	141.5	58	-62
20400	35.8	-36.5	30.9	-30.6	-12.0	+9.3	-143.1	141.0	57	-61
20300	35.7	-36.4	30.8	-30.5	-12.2	+9.4	-142.6	140.5	57	-61
20200	35.6	-36.3	30.8	-30.5	-12.5	+9.6	-142.1	139.9	56	-61
20100	35.5	-36.3	30.7	-30.4	-12.7	+9.7	-141.6	139.4	56	-60
20000	35.4	-36.2	30.6	-30.3	-12.8	+9.9	-141.1	138.9	56	-60
19900	35.3	-36.1	30.6	-30.3	-13.0	+10.0	-140.7	138.3	55	-60
19800	35.2	-36.1	30.5	-30.2	-13.2	+10.1	-140.3	137.8	55	-60
19700	35.1	-36.0	30.4	-30.1	-13.4	+10.2	-139.9	137.3	55	-60
19600	35.0	-35.9	30.3	-30.0	-13.6	+10.3	-139.4	136.7	54	-59
19500	34.9	-35.8	30.3	-30.0	-13.7	+10.4	-139.0	136.2	54	-59
19400	34.8	-35.8	30.2	-29.9	-13.9	+10.5	-138.5	135.7	53	-59
19300	34.7	-35.7	30.1	-29.8	-14.0	+10.6	-137.9	135.2	53	-58
19200	34.6	-35.6	30.0	-29.7	-14.2	+10.7	-137.3	134.7	53	-58
19100	34.5	-35.5	30.0	-29.6	-14.3	+10.9	-136.7	134.2	52	-58
19000	34.4	-35.4	29.9	-29.6	-14.4	+11.0	-136.2	133.6	52	-57
18900	34.3	-35.4	29.8	-29.5	-14.6	+11.1	-135.6	133.1	52	-57
18800	34.2	-35.3	29.7	-29.4	-14.7	+11.2	-135.1	132.5	51	-57
18700	34.0	-35.2	29.7	-29.3	-14.8	+11.3	-134.5	131.9	51	-56
18600	33.9	-35.1	29.6	-29.2	-14.9	+11.3	-134.0	131.3	50	-56
18500	33.8	-35.0	29.5	-29.2	-15.1	+11.4	-133.4	130.7	50	-56
18400	33.7	-34.9	29.4	-29.1	-15.2	+11.5	-132.9	130.1	50	-56
18300	33.6	-34.8	29.3	-29.0	-15.3	+11.6	-132.3	129.5	49	-55

Table F(i)

CHARGE 9 - SHELL 77B

Basic Data

Altitude Sea Level

1	2	3	4	5	6	7	8	9	10
Range	Firing Table elevation	Fuze setting	Correction to Fuze Setting to change Height of Burst by Down 10M	Effect on Range for increase of 1 mil in Firing Table elevation	Fork	Time of flight	Correction to Bearing for drift	Correction to Bearing for 1 knot cross wind	Correction to "Corrector" to change Height of Burst by Down 10M
(X)	(A_E)	(FS)	$\left(\frac{\Delta_C FS}{-10M Y_b}\right)$	$\left(\frac{\Delta_{EF} X}{+1 \text{mil} A_E}\right)$	(f)	(t)	$(\Delta_C A_d)$	$\left(\frac{\Delta_C A_B}{1KT W_Z}\right)$	
M	mil			M	mil	S	mil L	mil	points
18200	1188.2			-31.0	-9	105.7	74.1	1.41	
18100	1191.4			-31.3	-9	105.9	74.8	1.42	
18000	1194.6			-31.8	-9	106.0	75.5	1.43	
17900	1197.7			-32.0	-9	106.2	76.3	1.44	
17800	1200.8			-32.3	-9	106.3	77.0	1.45	
17700	1203.8			-32.8	-8	106.4	77.7	1.46	
17600	1206.8			-33.3	-8	106.6	78.4	1.47	
17500	1209.8			-34.1	-8	106.7	79.1	1.48	
17400	1212.8			-34.1	-8	106.9	79.8	1.49	
17300	1215.7			-34.4	-8	107.0	80.5	1.50	
17200	1218.6			-34.7	-8	107.1	81.3	1.51	
17100	1221.5			-34.7	-8	107.2	82.0	1.52	
17000	1224.4			-35.3	-8	107.4	82.7	1.53	
16900	1227.2			-36.6	-7	107.5	83.5	1.54	
16800	1230.0			36.6	-7	107.6	84.2	1.56	
16700	1232.7			-36.6	-7	107.7	85.0	1.57	
16600	1235.5			-36.6	-7	107.9	85.7	1.58	
16500	1238.2			-36.9	-7	108.0	86.5	1.59	
16400	1240.9			-37.2	-7	108.1	87.2	1.60	
16300	1243.5			-38.3	-7	108.2	88.0	1.61	
16200	1246.2			-38.3	-7	108.3	88.8	1.62	
16100	1248.8			-38.6	-7	108.4	89.5	1.64	
16000	1251.4			39.8	-6	108.5	90.3	1.65	
15900	1253.9			-39.8	-6	108.6	91.1	1.66	
15800	1256.5			-39.8	-6	108.7	91.9	1.67	
15700	1259.0			-39.8	-6	108.8	92.7	1.69	
15600	1261.5			-40.2	-6	108.9	93.5	1.70	
15500	1264.0			-40.2	-6	109.0	94.3	1.71	
15400	1266.4			-41.4	-6	109.1	95.1	1.73	
15300	1268.8			-41.4	-6	109.2	95.9	1.74	
15200	1271.2			-41.4	-6	109.3	96.7	1.75	
15100	1273.6			-42.7	-6	109.4	97.5	1.77	
15000	1276.0			-43.1	-5	109.5	98.3	1.78	
14900	1278.3			-43.1	-5	109.6	99.1	1.80	
14800	1280.7			-44.0	-5	109.7	100.0	1.81	
14700	1283.0			-44.0	-5	109.8	100.8	1.83	
14600	1285.2			-44.0	-5	109.9	101.7	1.84	
14500	1287.5			-44.5	-5	109.9	102.5	1.86	
14400	1289.7			-45.0	-5	110.0	103.3	1.87	
14300	1291.9			-46.5	-5	110.1	104.2	1.89	

Table F(ii)

CHARGE 9 - SHELL 77B

Corrections to range for non-standard conditions $\Delta_C X$

Altitude Sea Level

1	2	3	4	5	6	7	8	9	10	11
Range	1 M/S Muzzle Velocity (1 M/S V_O)		1 Knot Ballistic Wind (1 KT W_B)		1 % Ballistic Air Temperature (1% T_B)		1% Ballistic Density or Drag Coefficient (1% D_B)		1□ Projectile Weight (1□ m_{pj})	
(X)	Decrease (-)	Increase (+)	Head (W)	Tail (W)	Decrease (-)	Increase (+)	Decrease (-)	Increase (+)	Decrease (-)	Increase (+)
M	M	M	M	M	M	M	M	M	M	M
18200	33.4	-34.7	29.3	-28.9	-15.4	+11.6	-131.8	128.8	49	-55
18100	33.3	-34.6	29.2	-28.8	-15.4	+11.7	-131.3	128.2	48	-55
18000	33.2	-34.5	29.1	-28.7	-15.5	+11.7	-130.8	127.6	48	-55
17900	33.1	-34.4	29.0	-28.7	-15.6	+11.8	-130.2	127.0	48	-54
17800	32.9	-34.3	28.9	-28.6	-15.6	+11.8	-129.7	126.3	47	-54
17700	32.8	-34.1	28.8	-28.5	-15.7	+11.9	-129.1	125.7	47	-54
17600	32.7	-34.0	28.7	-28.4	-15.7	+12.0	-128.4	125.1	47	-53
17500	32.6	-33.9	28.7	-28.3	-15.7	+12.0	-127.7	124.5	46	-53
17400	32.4	-33.7	28.6	-28.2	-15.7	+12.1	-127.0	123.8	46	-53
17300	32.3	-33.6	28.5	-28.1	-15.7	+12.1	-126.3	123.2	46	-52
17200	32.2	-33.4	28.4	-28.0	-15.7	+12.2	-125.6	122.6	45	-52
17100	32.1	-33.3	28.3	-27.9	-15.7	+12.3	-124.8	122.0	45	-51
17000	31.9	-33.1	28.2	-27.8	-15.7	+12.3	-124.1	121.3	45	-51
16900	31.8	-33.0	28.1	-27.7	-15.7	+12.3	-123.4	120.7	45	-50
16800	31.6	-32.8	28.0	-27.6	-15.7	+12.3	-122.7	120.1	44	-50
16700	31.5	-32.7	27.9	-27.4	-15.7	+12.4	-121.9	119.4	44	-50
16600	31.4	-32.5	27.8	-27.3	-15.7	+12.4	-121.2	118.8	44	-49
16500	31.2	-32.4	27.7	-27.2	-15.7	+12.4	-120.4	118.1	43	-49
16400	31.1	-32.2	27.6	-27.1	-15.7	+12.4	-119.6	117.5	43	-48
16300	30.9	-32.1	27.5	-27.0	-15.6	+12.5	-118.7	116.8	43	-48
16200	30.8	-31.9	27.4	-26.9	-15.6	+12.5	-117.8	116.2	43	-47
16100	30.7	-31.7	27.3	-26.7	-15.5	+12.5	-116.9	115.5	42	-46
16000	30.5	-31.5	27.2	-26.6	-15.5	+12.5	-116.1	114.8	42	-46
15900	30.4	-31.4	27.1	-26.5	-15.4	+12.5	-115.2	114.1	42	-45
15800	30.2	-31.2	27.0	-26.4	-15.4	+12.5	-114.4	113.4	41	-45
15700	30.1	-31.0	26.9	-26.2	-15.3	+12.5	-113.7	112.5	41	-44
15600	29.9	-30.9	26.8	-26.1	-15.3	+12.5	-112.9	111.7	40	-44
15500	29.7	-30.7	26.7	-26.0	-15.2	+12.4	-112.2	110.9	40	-44
15400	29.6	-30.5	26.5	-25.8	-15.2	+12.4	-111.4	110.0	39	-43
15300	29.4	-30.3	26.4	-25.7	-15.1	+12.4	-110.6	109.2	39	-43
15200	29.2	-30.2	26.3	-25.5	-15.0	+12.3	-109.8	108.4	39	-42
15100	29.1	-30.0	26.1	-25.3	-15.0	+12.3	-109.0	107.6	38	-42
15000	28.9	-29.8	26.0	-25.2	-14.9	+12.2	-108.2	106.7	38	-42
14900	28.7	-29.6	25.9	-25.0	-14.8	+12.2	-107.4	105.9	37	-41
14800	28.6	-29.4	25.7	-21.8	-14.7	+12.2	-106.6	105.1	37	-41
14700	28.4	-29.2	25.6	-24.7	-14.6	+12.1	-105.7	104.2	37	-40
14600	28.2	-29.0	25.4	-24.5	-14.6	+12.1	-104.8	103.4	36	-40
14500	28.0	-28.8	25.3	-24.3	-14.5	+12.0	-103.9	102.5	36	-39
14400	27.9	-28.6	25.1	-24.1	-14.4	+12.0	-103.0	101.6	35	-39
14300	27.7	-28.5	24.9	-23.9	-14.3	+11.9	-102.1	100.7	35	-38

Table F(i)

CHARGE 9 - SHELL 77B

Basic Data

Altitude Sea Level

1	2	3	4	5	6	7	8	9	10
Range	Firing Table elevation	Fuze setting	Correction to Fuze Setting to change Height of Burst by Down 10M	Effect on Range for increase of 1 mil in Firing Table elevation	Fork	Time of flight	Correction to Bearing for drift	Correction to Bearing for 1 knot cross wind	Correction to "Corrector" to change Height of Burst by Down 10M
(X)	(A_F)	(FS)	$\left(\Delta_C FS / -10M Y_b\right)$	$\left(\Delta_{FE} X / +1 m A_E\right)$	(f)	(t)	$(\Delta_C A_d)$	$\left(\Delta_C A_B / 1KT W_Z\right)$	
M	m			M	m	S	mL	m	points
14200	1294.1			-46.5	-5	110.2	105.1	1.91	
14100	1296.3			-46.5	-5	110.3	105.9	1.92	
14000	1298.4			-46.5	-5	110.3	106.8	1.94	

Table F(ii)

CHARGE 9 - SHELL 77B

Corrections to range for non-standard conditions $\Delta_C X$

Altitude Sea Level

1	2	3	4	5	6	7	8	9	10	11
Range	1 M/S Muzzle Velocity (1 M/S V_0)		1 Knot Ballistic Wind (1 KT W_R)		1 % Ballistic Air Temperature (1% T_B)		1% Ballistic Density or Drag Coefficient (1% D_B)		1 □ Projectile Weigh (1 □ m_T)	
(X)	Decrease (-)	Increase (+)	Head (W)	Tail (W)	Decrease (-)	Increase (+)	Decrease (-)	Increase (+)	Decrease (-)	Increase (+)
M	M	M	M	M	M	M	M	M	M	M
14200	27.5	-28.3	24.8	-23.7	-14.2	+11.8	-101.1	99.8	34	-38
14100	27.3	-28.1	24.6	-23.4	-14.1	+11.8	-100.1	98.9	34	-37
14000	27.1	-27.9	24.4	-23.2	-13.9	+11.7	-99.2	97.0	33	-36

Explanatory Notes on 155mm BOFORS Range Tables (RT)

Basic Data (Table F (i))

Column No.	Description
1	Range (X) : Range in m, being aimed
2	Firing Table Elevation (A_E)· Elevation in mils (360 deg = 6400 mils) to be set on the gun to achieve range at col. 1.
3	Fuze setting (FS) setting to be done on the fuze so that it functions(bursts during flight) at the time mentioned at col. 4 (in case time fuze being used)
4	Corrn to fuze setting(FS) to change height of burst by down 10 m.
5	Effect on range, in m, for increase of 1 mil in firing table elevation
6	Fork (also known as short bracket) : when a large no of shells are fired at the MV and elevation in a series (i.e. in quick succession) they will exhibit some dispersion pattern, which (supposedly) follows normal statistical distribution. The x(range) and y (cross range) co-ordinates of fall of shot are treated separately as independent variables and the standard deviation σ_x & σ_y in x & y resp. are obtained 50 % zone in range and cross range is defined as the length zone spread equally around the mean point of impact (more commonly known as MPI) of a series of rounds fired which may contain 50% of the rounds if a large no. of rounds are fired. 50% Zone = 1.349 σ_x (or σ_y as applicable) Half the 50% zone is known as probable error. i.e 50% zone consists of one probable error on either side of the MPI. Fork : the change in elevation required to produce a range change equivalent to 4 x range probable errors. This parameter is mainly used for range safety purposes.
7	Time of flight : the time taken by the projectile to travel from the moments it leaves the gun barrel to the point of impact.
8	Correction to Bearing for drift· The angular correction to be applied to the line of fire called bearing (bearing is the angle measured clockwise from local North) to compensate for the deviation of the shell from the line of fire towards right due to its right hand spin. Most of the artillery shell are gyro. stabilized hence have an eq^{-m} total angle of attack (called yaw of repose in ballistics) towards right and upwards wrt the velocity vector resulting in the deviation of projectile to right of line of fire.
9	Correction to bearing for 1 knot cross wind : corrn to be provided to the bearing of line of fire to nullify the deviation of the shell due to constant cross wind of 1 Knot magnitude through out the trajectory at all heights. The correction is – ve if the crosswind is blowing from Lt to Rt i.e. + ve cross wind In practice the wind will never be constant. at all heights. A weighted mean value (wrt heights) is obtained called as equivalent constant wind (ECW) A wind of quantity ECW will produce the same effect as that of the actual-prevailing wind. Hence for the same prevailing wind conditions ECW values will be different for different ranges due to the difference in the vertex heights. Same thing is applicable in case of range i e. head or tail wind The measured wind data at different is first resolved in to range and cross wind components at all heights. Equivalent constant winds for range and cross components are calculated and used separately.
10	Corrn to "corrector" to change height of burst by down 10 m.

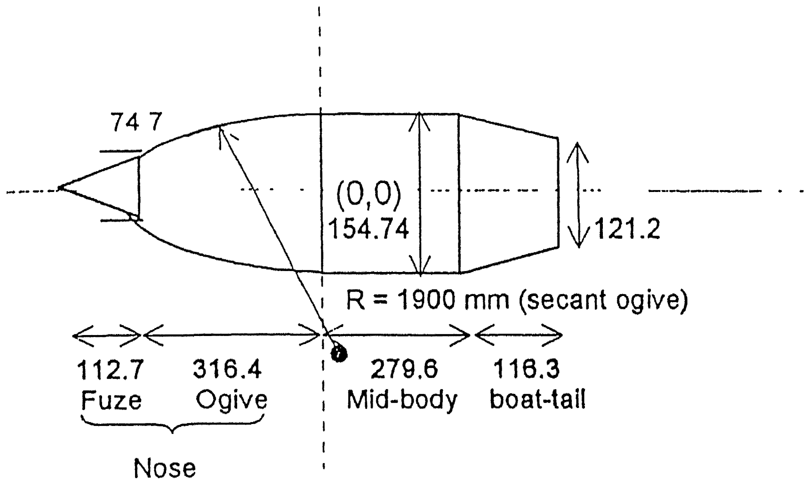
Correction to range for non-standard conditions (Table F (ii)) ;

Column No.	Description
1	Range (X) : Range in m, being aimed
2 & 3	Correction to range in m for a change in muzzle velocity (MV) by 1 m/s If the MV increases then the range also increases, hence the correction to range for increase in MV is -ve
4 & 5	Correction to range in m for a range wind of magnitude 1 knot A tail wind is taken as +ve & head wind –ve If it is tail wind then the range increases, hence the correction to range is -ve
6 & 7	Correction to range in m for a change in ballistic air temp by 1% of the standard . The RT is compiled for some standard meteorological conditions, which defines the air temp , pressure and density at various altitudes. In case of 155mm RT ICAO standard atmosphere is assumed Air is assumed to be dry in ICAO but in practice it may have some moisture. Also the temp and press may not be the same as the standard Ballistic air temp. (BAT) : BAT is the temp. of the dry air having an equal density as that of moist air. The correction for BAT only takes care of the changes in range on account of the changes in Mach No. due to variation in temp. (recall that acuastic velocity = $\sqrt{(\Upsilon RT)}$) and not the variation in density due to changes in temp. This is also termed as elasticity effects.
8 & 9	**Correction to range in m for a change in ballistic density (ρ) or change in drag coefficient (C_D)by 1 %. If ρ or C_D increases then the range decreases, hence the correction to range is +ve.** Density at different altitudes will be different, however a single weighted mean value is obtained (in the same manner in which wind is treated).
10 & 11	**Projectile weight : Correction to range for a projectile whose weight differs from the standard weight by 1 unit (□ m_{pj} : □ marking is done on the shells after manufacture for there identification in terms of weight class. One such square means that the shell weight differs from the std. by 1 unit). The method of marking is different for different equipments.** The effect of change in shell weight is two fold: first the MV reduces resulting in the reduction in range and second the carrying capacity increases (Drag / Mass decreases i.e. less deceleration than the normal) resulting in increase in range. The net effect could be either way. For shell HE 77B 1(□ m_{pj})= 0.450 kg

Explanation about "Ammunition Data(cont.)" on page (x)

Shell weight influence:-

The relationship given there viz. Δ Mv = A2 x Δ 100 g shell weight,
Gives the change(decrease) in muzzle velocity (Mv) due to 100 gm change(increase) in the shell weight for various charges (M3A1 3G etc are designated propellant charges with fixed qty of propellant. Each charge corresponds to a certain Mv. It can be seen that the shell influence is increasing as the Mv increases (from top to bottom)



Total Length = 825 mm

CG Location = 533 mm from Nose-tip

Moment of Inertia·

Axial, I_{XX} = 0.146 kg-m^2

Transverse, I_{YY} = 1.709 kg-m^2 (through an axis passing through CG)

Twist of Rifling(n) = 1 in 20 calibres. $\Rightarrow$ the shell will complete one full rotation (360^0), in side the gun barrel by the time it does a translation or linear travel of 20 calibres i.e. 20 * 0.155 = 3.1 m.

Thus launch spin = Mv / (n.d) = 818 / (20 * 0.155) = 263.87 rev/s = 15832 rpm

Fig A1. Schematic of 155 mm Bofors Shell HE 77B

Appendix B

FEED FORWARD NEURAL NETWORK

The back propagation network consists of one input layer, one output layer and one or more hidden layers. There is no theoretical limit on the number of hidden layers but typically there is just one or two. Some work has been done [14], which indicates that minimums of four layers (three hidden layers and one output layer) are required to solve problems of any complexities. Each layer is fully connected to the succeeding layer (standard connection).

There are as many neurons in the input layer as there are inputs, and likewise with the output layer. The number of layers and the number of neurons in the hidden layer(s) must be determined by trial and error. There is no quantifiable best answer to the layout of the network for any particular application. There are only general rules picked up over time and followed by most researchers and engineers applying various architectures to their problems.

Rule 1. As the complexities in the relationship between the input data and the desired output increases, the number of processing elements in the hidden layer should increase.

Rule 2. If the process being modeled is separable into multiple stages, then additional hidden layers may be required. If the process is not separable into stages, then additional layers may simply enable memorization and not a true general solution.

Rule 3. The amount of available training data sets an upper bound for the number of processing elements in the hidden layer. To calculate this upper bound, use the number of input-output pair examples in the training set and divide that number by the total number of input and output processing elements in the network. Then divide the result again by a scaling factor between five and ten. Larger scaling factors are used for noisy data.

Extremely noisy data may require a factor of twenty or even fifty. Very clean input data with an exact relationship to the output might allow the factor to be dropped to around two.

Cross-Validation and Overtraining

One approach to avoid over-training of the network is to estimate the generalization ability during training and stop when it begins to decrease. The essence of back-propagation learning is to encode an input-output relation, presented by a set of data, with a multiplayer perceptron well trained in the sense that it learns enough about the past to generalize to the future. The simplest method is to randomly partition the data set into a training set and a test (validation) set. From the training set, a validation subset, which are typically 10 to 20 percent of the training set is set aside. The motivation here to validate the model on a data set different from the training set that is used for selecting the architecture of the network. The training set is used to modify the weights, the validation set is used to estimate the generalization ability. The architecture of the network is varied till the training set results in MSE less than the prescribed value ε. This architecture is now tested on the test data (which can be one or more) and if the MSE is of the order of 2ε, the architecture is accepted to yield the desired neural model and assumed to be capable of predicting required output for inputs not seen earlier by the network.

Another way of avoiding over-training is to limit the ability of the network to take advantage of spurious correlation in the data. Over fitting is thought to happen when the network has more degrees-of-freedom (the number of weights, roughly) than the number of the training samples when there are not enough examples to constrain the network.

Even though it may give exactly right output at the training points, it may be very inaccurate at other points. An example is a higher order polynomial fitted through a small number of points.

Sufficient Training Set Size For a Valid Generalization

Generalization is influenced by three factors: i) the size and the efficiency of the training set, ii) the architecture of the network, and iii) the physical complexities of the problem at hand. Clearly, we have no control over the last factor, i.e., the physical complexity. We have already discussed the choice of architecture based on training and test data. Once the architecture of the network is fixed, then the size of training set can be derived as follows.

Let M denote the total number of hidden layer computation nodes. Let W and N be the total number of synaptic weights and the number of random examples used to train the network respectively. Let ε denote the fraction of error permitted on test. Then, according to Baum and Haussler, [2] the network will almost certainly provide generalization provided the following two conditions are met.

(a) The fraction of error made on the training set is less than $\varepsilon/2$.

(b) The number of examples(N) used in the training is

$$N \geq 32 \frac{W}{\varepsilon} \ln \frac{W}{\varepsilon}$$

where W is the total number of synaptic weights.

Ignoring the logarithmic factor, taking first order approximation, the number of training examples is directly proportional to the number of weights in the network and inversely proportional to the accuracy parameter ε. Then, $N > \frac{W}{\varepsilon}$.